

COMPARATIVE ATTRIBUTES OF HIGH-FREQUENCY MARKET MAKING ALGORITHMS

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Abstract: Comparison and evaluation of existing high-frequency market making algorithms which provide continuous bid and ask liquidity in financial markets is very difficult, because algorithm effects directly influence behaviour of trader and that influences back the behaviour of algorithm (a chicken-egg problem). The paper presents representative sample of algorithms, focuses on their differences and proposes a set of areas which are important for comparison due to market specifics.

Keywords: market making, high-frequency, comparison of algorithms, bid and ask liquidity

1. INTRODUCTION

There are several markets, local or international, where securities can be traded. Such a security market provides a venue for transactions between buyers and sellers. One of the main properties is efficiency measured by its liquidity – how quickly the offer can be satisfied. To ensure good liquidity, traditionally we have special firms on a market, the market makers, which are quoting bid and ask prices (explained later) for some specific assets and its securities.

The market maker is offering to buy or sell a security almost any time, but the price for it and for a business risk is balanced by a lower selling prices (called a bid price) and higher purchase price (called a ask price). The difference between them is called a spread and it's obviously one of the main parameters which have direct influence on a market's liquidity. The spread is flexible and has generally three major components: order processing costs, inventory-holding costs and adverse selection costs. The third part means influencing by asymmetric information risk between the market maker and informed traders. On the other hand, thank to wide using of direct market access there is a large range of new investors implementing high-frequency trading algorithms. The consequence is that competition among liquidity providers is renewed, effective market spreads are reduced and therefore indirect costs for final investors are reduced also (empirical studies in [1]).

The bid and ask orders together with limit asset's quantity are written into a limit order book on a FIFO principle – a trader wants to sell or buy securities and first offer with the suitable price will be executed before the next one; if the quantity defined in the offer is used up, then the next offer comes. More about how the limit order book influences algorithm can be found in [2].

The holding time can divide the trading strategies to buy-and-hold and high-frequency. The first one means trader is holding bought securities from hours to even years. In comparison the high-frequency trading holds for milliseconds or even microseconds. Current market making algorithms fall under the high-frequency and comparison of these algorithms is studied over the world. This paper is joining the research and it's an introduction to a following deeper exploring.

2. SELECTION OF REPRESENTATIVE ALGORITHMS

A market maker faces on the market to an adverse selection problem caused by asymmetric information: different traders on the market have different information about the fundamental value of

security. In response, the market maker can take several actions to prevent the loss; for example, increase the bid-ask spread or changing quotes volumes to discourage further trades on the same side and encourage trades on the other side. The major of academic market making strategies are using the first one to risk and loss reduction, so as we in this paper.

Various market making algorithms have been proposed in the literature and there is an effort to evaluate their benefits and disadvantages in a systematic manner in the last few years. The paper presents a selection of representative strategies and it is focusing on their basic idea and differences between them. Selected strategies are current algorithms from academic environment and private sector is using their proprietary modifications. This fact should have no effect to our objective to prepare set of comparative approaches, because methodology should be universal.

2.1. LMSR STRATEGY

The strategies based logarithmic market scoring rule (LMSR) can be considered as the standard. The original algorithm and its variants suffer from an inability to react rapidly to jumps in traders' beliefs. The LMSR market makers are loss-making and need to be subsidized (described in [5]).

We can find whole technical description in [4] and [5]. Basic idea is the market maker will take the opposite side of any order at a price specified by the market maker. This price depends on a parameter b and the market maker's current inventory q_t , where t means the order of a specific time tick. The bid-ask spread $\delta(Q)$ for quantity Q is the difference between the average price paid for buying Q securities versus selling Q them and can be count as formula (1) shows.

$$\delta(Q) = \frac{b}{Q} \ln\left(\frac{\cosh q_t / b + \cosh Q / b}{2 \cosh^2 q_t / 2b}\right) \quad (1)$$

The parameter b is the only free parameter in the LMSR strategy. It bounds the loss, controls market liquidity and also controls how adaptive the strategy is. If b is small, the strategy is very adaptive and taking small loss, but spreads are large and it leads to the low market liquidity.

Advantages of the LMSR strategies are deterministic behaviour of amount of loss it can suffer, a small number of parameters (only one, but this can be considered as a disadvantage too) and when the participants are acting rational and learn from prices, the LMSR strategy leads to the rational expectations equilibrium price (described in [6]).

Disadvantages of the LMSR strategies are quite frequently running at a loss and, in the connection with it, a slow adaptation on large market jumps. How we mentioned above, the only free parameter b which affects multiple components of the strategy behaviour can be consider as a disadvantage too, because on a market with frequent asymmetric fluctuations and with large random jumps is convenient setting of the parameter b really complicated.

2.2. INFORMATION BASED STRATEGY

A market maker who arrives at the market faces an adverse selection problem – informed traders in the market have mostly more information about the expected price movement of the underlying security and the market maker lose money in trades with them. Basically, the strategy tries to find and use the proportion of the informed and uninformed traders which is based on the bid-ask quotes and calculated by Glosten Milgrom model (the model is described in [3]). The final enhancement is a supervised learning procedure optimizing behaviour from the high-frequency data.

The strategy computes the proportion of informed and uninformed traders by the GM model (or the extended GM model, if we are considering a minimal transaction cost). When there is the higher proportion of the informed traders than can be beneficial, submitting of limit orders to the market stops until number of informed traders goes down to a reasonable value. The trading agent is learning and influencing its state by new values due to the proportion of informed traders.

If we look closer and compare this strategy to previous one, the LMSR strategy is adaptive, but non-convergent; the information based strategy is convergent, but only slowly adaptive, potentially incurring a large loss. On the other hand, the information based strategy offers surely quite a good liquidity and a short spread.

2.3. BMM STRATEGY

Bayesian Market Maker strategy (BMM) provides liquidity by adapting its spread based on its level of uncertainty about the true value. The BMM strategy is based on the LMSR strategy and it uses two processes: the first is focussing on the trade price and its size; the second is evaluating jumps whether they occurred. This allows it to achieve small spreads in equilibrium-like states with better adaptation to market jumps.

One of the main parameters is the size of the windows which is historically mapping behaviour of the market security. The result from this window calculation is used for quicker adapting for market jumps. On the other hand, this adaptive speed is paid by starting time of an adaptation process. The detailed description of whole process can be found in [5].

The BMM strategy compared with the LMSR strategy provides following benefits: it adapts quickly and generally does not lose money, while providing a liquid market. It has better convergent behaviour at equilibrium than the LMSR strategy. The experiments show that the BMM strategy can adapt rapidly to changing valuations in the trading population and has low stable spreads at equilibrium which implies the better market liquidity. But when many large jumps occur really fast on the market, a partial loss of BMM strategy can be significant higher than with LMSR. The BMM strategy is not loss bounded, and there is some risk of higher substantial loss.

2.4. OTHER SUITABLE ACADEMIC STRATEGIES

There are plenty of the market making strategies and those described above are not the only ones considered for this paper. A small part on the preparation on the property list should be attributed to the Inventory strategy and Symmetric inventory strategy from [2].

3. COMPARATIVE PITFALLS AND SUGGESTIONS

3.1. AFFECTING THE MARKET ENVIRONMENT

As we mentioned above, comparison of market making algorithms is a difficult topic. These algorithms provide a continuous bid and ask liquidity in financial markets, so they are directly influencing the market. You cannot easily simulate an algorithm on the historical data, because if the algorithm would be there, there is a high probability that this data should look different. That's a classic chicken-and-egg problem, because market makers are bringing liquidity to create more liquidity.

Another unknown variable would be a percentage of success in competition with other informed traders who are quoting the bid and ask limit orders into the limit order book. It has two consequences: first, competition for a better offer, and also better place in the queue in the limit order book. To the second point, experiences showed the queue of the limit orders can be quite long and when the request comes on line, it can be after some time (amount of time depends on the market liquidity). When the market maker counts in microseconds, any slowdown can influence its return. Because the market and a price of security can change really fast, the back position in the queue can cause that at the time of the execution it is disadvantageous to trade and it is better to cancel the order. The formula (2) shows the calculating of market maker's profit $P(1)$ when it buys one security in t_1 and sells in t_2 as the difference between the spread S and change of the ask price.

$$P(1) = S_{t_1} - (ask_{t_1} - ask_{t_2}) \quad (2)$$

3.2. THE STRATEGY COMPARISON PROPERTIES

Very important idea for a further work is that there are different markets with different parameters, different legislative environments (for example a minimal spread), different securities and different traders. If we are focusing on the market environment we can find the following key properties

- a proportion of informed and uninformed traders (see the Glosten Milgrom model [3]),
- a structure of fees, like an order execution fee, exchange fees, trade clearing costs etc.
- a proportion of buy-and-hold and high-frequency traders.

We can try to specify the key properties of the market making strategies. The obvious ones are

- an influence on market liquidity, which is mostly measured by a width of bid-ask spread (see the Roll model [3]),
- adaptability, which means a speed of cope after the market jump or shock (when the offering bid and ask price is “one-sided”),
- profitability.

Properties which are found on a closer look on market making strategies and a trading process are

- a speed of convergence to a real security value,
- a speed of evaluation,
- a count of “out the money” positions (not selling with higher price than buying),
- a stability of behaviour at an equilibrium,
- a maximal or a partial loss,
- a market next-step forecast.

The last type of influence which can be used to increase a profit or decrease a loss is a detection of market environment. If the strategy has deeper insight into the market, such the technique can be used as “stop-buy” (or “stop-sell”), when it is not convenient, or optimize the time of orders for the better position in the limit order book.

3.3. TECHNICAL DESIGN OF COMPARISON

There are several technical designs for a market making strategy evaluation. Each of them has own pitfalls and all of them give only a partial and probabilistic answer. To obtain a sufficient basic idea about strategy behaviour on the target market it is appropriate to use a combination of them. As the leading representative we can consider the following designs:

- Mathematical analysis – the main issue is the human traders can behave randomly. We can enhance the results by the statistical data from the target market, but the problem described in the section 2.1 still occurs. This is a good starting position, but until there is a large group of human traders, the mathematical analysis wouldn't be enough.
- Simulation on the historical or actual market data – the issue from the section 2.1 is still there. Like the mathematical analysis this is a good starting position to evaluate the strategy and it represents the situation with a small market share and a small market influence.
- Agent simulation – how to define the agent behaviour to make it act like the human? We can consider a weaker condition: the market composed of these agents should have the same behaviour as composed of the real market traders. The problem is almost the same like with mathematical analysis. We can enhance the result by statistical properties of the target market, but the same problems as described the section 2.1 still occur.

- Experiments with human subjects – three main challenges with the human trading experiment are: (1) how to create an appropriate group of people; (2) the same group of traders cannot be used first in an experiment with one strategy and then in an identical experiment with second strategy; (3) the same experiment cannot be run on two separate groups of traders with the different strategies in each group due to the high variability in human traders and small sample sizes. All these weaknesses can be statistically reduced by increasing the number of test group members and sample size, but still it could be only a simulation. More about the human experiment and a nice example can be found in [5].

4. CONCLUSION

The paper summarizes the current state of research in the field of high-frequency market making algorithms and defines a set of convenient parameters for the strategies' comparison. There aren't included last trends as the "hiding" of large orders by its time and place decomposition, sending the false and processing-time-consuming signals, an artificial temporary increase of a price in the detection of large orders heading to the market with immediate subsequent sale back and so on. The research in the paper is focussing on the properties of the market making algorithms instead of the pure business strategies. In the final part we summarized the technical test designs and we tried to found the disadvantages of the each type of strategy evaluation.

It is mentioned as an introduction to further research which is deeply inspecting these areas due to practical use for the strategy evaluation. The next step should be a publication of the paper to each of the comparative areas and as the final step there should be a publication of whole comparative methodology. The research primarily concerned with preparing suitable historical or agents' simulation for each specific comparative area, starting with unconventional areas such as the count of "out the money" positions or the market next-step forecast.

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