TUNING OF COMPLEMENTARY FILTER ATTITUDE ESTIMATOR USING PRECISE MODEL OF MULTICOPTER

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Abstract: This contribution deals with attitude estimation algorithm based on measuring angular rate, earth magnetic field and accelerations. At first, two main methods usually used for attitude estimation are described. Subsequently the particular form of complementary filter attitude estimator is shown. Then the tuning of algorithm parameters using MATLAB environment is described. For this, precise model of multicopter, which was made in Simulink environment, was used. Testing trajectory along with simulated sensor and true data were made using above mentioned model, so the comparison through RMSE (Root Mean Square Error) value is available.

Keywords: Multicopter, Attitude Estimation, Kalman Filtering, Complementary Filter

1. INTRODUCTION

Attitude estimation is a crucial part of any autonomous aerial system. Usually a typical attitude measuring sensor consists of cheap tri-axis MEMS gyroscopes, accelerometers and magnetometers. All these sensors are noisy and highly biased.

The gyroscopes measure three components of angular rate with respect to inertial frame expressed in so called body frame (linked with examined object). If the initial attitude is known, the time evolution of attitude can be computed by integrating the angular rate data. Because of the noise and the bias which is more or less present in all angular rate sensors the error grows unboundedly in time. Using cheap MEMS gyroscopes, the attitude computed only by integration is useless after few tens of seconds.

The accelerometers measure specific force acting on the examined object. If the object is not moving (or uniformly moving), the accelerometers measure Earth gravitational field. This information can be used for computing Roll and Pitch Euler angles directly (Euler angles – one of the possible attitude representations [1]). If any acceleration different from gravitational acts on the examined body, the information is useless during this period.

The last sensor, the magnetometer measures magnetic field, if there is no local source of magnetic field, this sensor measures Earth magnetic field which in small time horizon provides constant vector. If we know magnetic vector in reference frame, then by measuring magnetic vector in body frame we get limited attitude information (no direct Euler angle can be computed). This information is relevant unless the magnetic field is disturbed by local magnetic sources.

Each of above mentioned type of sensors provides some kind of information regarding attitude. However this information alone is not usable for long time attitude estimation with bounded error. Usually these three types of sensors are used together and some sophisticated algorithm uses the advantages of each sensor and combines the information to provide the best attitude estimate. The most used attitude estimation methods utilize some special type of Kalman filter [2] or Complementary filter [3]. Each of these methods will be briefly introduced in the following chapter.

2. METHODS USED FOR ESTIMATION – BASIC PRINCIPLES

In this chapter only the basic principles of each method will be described. These principles and characteristics are more or less general and have no limitation on attitude estimation.

2.1. COMPLEMENTARY FILTER

Complementary filter is a filtering technique in frequency domain. Two or more sensors, which provide some state variables of measured system, are considered as an input. From each sensor, only a part of frequency spectrum is used and all sensors together cover all spectrum. This means that one sensor complements other in frequency domain, thus the name Complementary. The block scheme of complementary filter is depicted in Figure 1.

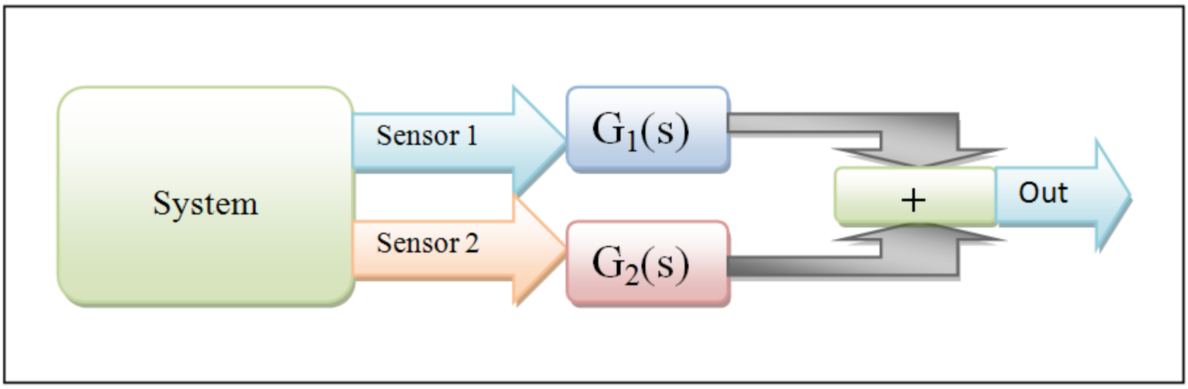

Figure 1: Principle of Complementary filter.

If we have two sensors of the same state variable this condition for filter G_1 and G_2 should be satisfied:

$$G_1(s)+G_2(s)=1 \tag{1}$$

The Complementary filter is widely used mainly because of the ease of implementation and for its simplicity (only one parameter – crossover frequency is required for two sensor case).

2.2. KALMAN FILTER

Kalman filter is a well known estimation technique developed in 1960 [4]. It is primarily developed for estimating the state of linear systems with additive Gaussian white noise with noisy measurements. If we know the characteristic of all noises, the Kalman filter algorithm guarantee the optimality of its state estimate. The iterative discrete algorithm consists of two steps, the prediction step and the correction step. The individual steps of Kalman filter are:

Prediction step:

$$\mathbf{X}(k+1|k)=\mathbf{A}\cdot\mathbf{X}(k)+\mathbf{B}\cdot\mathbf{U}(k) \tag{2}$$

$$\mathbf{P}(k+1|k)=\mathbf{A}\cdot\mathbf{P}\cdot\mathbf{A}'+\mathbf{R} \tag{3}$$

Update step:

$$\mathbf{K}=(\mathbf{P}(k+1|k)\cdot\mathbf{C}')\cdot inv(\mathbf{C}\cdot\mathbf{P}(k+1|k)\cdot\mathbf{C}'+\mathbf{Q}) \tag{4}$$

$$\mathbf{X}(k+1)=\mathbf{X}(k+1|k)+\mathbf{K}(\mathbf{Z}(k)-\mathbf{C}\cdot\mathbf{X}(k+1|k)) \tag{5}$$

$$\mathbf{P}(k+1)=(\mathbf{I}-\mathbf{K}\cdot\mathbf{C})\cdot\mathbf{P}(k+1|k) \tag{6}$$

Where $\mathbf{X}(k)$ is a state vector at k-th iteration, $\mathbf{A}$ is the system matrix, $\mathbf{B}$ is the input matrix, $\mathbf{C}$ is the measurement matrix, $\mathbf{Z}$ is the vector of measurements, $\mathbf{P}$ is the system covariance matrix, $\mathbf{R}$ and $\mathbf{Q}$ are covariance matrices of system and measurement noises.

Since the original Kalman filter is only for linear systems some suboptimal adaptation for nonlinear systems were developed. The most common is EKF (Extended Kalman Filter) which uses first order Taylor expansion in every iteration. Regarding the attitude estimation, Kalman filter is more difficult to use than the complementary filter. There are more parameters to tune (system and measurement noise covariance matrices) and the whole algorithm is computationally very expensive and the implementation to the target device with microcontroller needs lot of effort in comparison with complementary filter.

3. COMPLEMENTARY FILTER ATTITUDE ESTIMATOR

If we take into account the principle of complementary filter along with characteristic of individual sensors mentioned in introduction section, it is beneficial to use only high frequency component of gyroscope sensor and low frequency component of the remaining sensor. The implementation of complementary filter can have different forms, but the basic principle of frequency filtering is still present. Hereafter mentioned algorithm is modified complementary filter attitude estimator from [3].

The rotation matrix is used for internal attitude representation. Rotation matrix is the only unique and non-singular attitude representation [1]. The only disadvantage is the number of elements - 9 (quaternion – 4, Euler angles - 3). There exist simple relations between all these attitude representations. The core of the algorithm is the discrete equation integrating the gyroscope sensor values using the rotation matrix *R*:

$$\mathbf{R}_{n+1} = \mathbf{R}_n \cdot (\mathbf{I} + \mathbf{\Omega}_n \cdot \Delta T) \tag{7}$$

$$\mathbf{\Omega} = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix} \tag{8}$$

Where ΔT is sampling period, n is iteration index, $\mathbf{I}$ is identity matrix and $\boldsymbol{\omega}$ is vector of angular rates (expressed in body frame). The information from accelerometer and magnetometer are passed to the core of the algorithm through so-called bias estimate:

$$\mathbf{b}(n) = k_p \cdot \mathbf{e}_n + k_i \sum_{i=1}^{n} \mathbf{e}_i \tag{9}$$

$$\mathbf{e} = k_a \cdot v \cdot \mathbf{e}_a + k_m \cdot \mathbf{e}_m \tag{10}$$

$$\mathbf{e}_a = (\mathbf{R}' \cdot \frac{\mathbf{g}_I}{\|\mathbf{g}\|}) \times (\frac{\mathbf{a}_B}{\|\mathbf{a}\|}) \tag{11}$$

$$\mathbf{e}_m = (\mathbf{R}' \cdot \frac{\mathbf{m}_I}{\|\mathbf{m}_I\|}) \times (\frac{\mathbf{m}_B}{\|\mathbf{m}_B\|}) \tag{12}$$

$$v = \exp\left(\frac{-(\|\mathbf{a}\| - g)^2}{\sigma} \right) \tag{13}$$

Where k_p, k_i, k_a, k_m and $\boldsymbol{\sigma}$ are parameters of complementary filter, $\mathbf{g}$ is gravitational acceleration vector, $\mathbf{a}_B$ is measured acceleration $\mathbf{m}$ is Earth magnetic vector, subscript *I* means reference value expressed in inertial frame and subscript *B* means measured value expressed in body frame. The terms in (11) and (12) are angular rate vectors expressed in body frame, which would lead to alignment of reference and measured vectors. Term (13) causes that the acceleration vector is used only when its magnitude is close to value of Earth gravitational acceleration, so it avoids using information from accelerometer when it is irrelevant. Information from magnetometer is considered to be relevant all the time (this assumption can be violated easily in indoor environment). As term

(9) is suggests direction of rotation leading to alignment (reference vectors with measured vectors) multiplied by constant it is clear that this vector (bias estimate) is passed back to the core algorithm:

$$\boldsymbol{\omega} = \boldsymbol{\omega}_B - \mathbf{b} \tag{14}$$

Where $\boldsymbol{\omega}$ is vector of angular rates from (8), $\boldsymbol{\omega}_B$ is measured angular rate by gyroscope and $\mathbf{b}$ is bias estimate from (9). Sum of all previous errors in (9) allows to have zero stable error because of the same principle as in I term in PID controller.

Since all computations are usually performed on computer, care has to be taken to rotation matrix. Rotation matrix is a special orthonormal matrix (all rows or columns are orthogonal and perpendicular vectors). This property is slightly violated each iteration. Without correction, this would lead to divergence of whole algorithm. One of the possible orthogonalization equations is [5]:

$$\mathbf{R} = \tfrac{3}{2}\mathbf{R} - \tfrac{1}{2}\mathbf{R}\cdot\mathbf{R}'\cdot\mathbf{R} \tag{15}$$

Equations (7)-(15) form the complete complementary filter algorithm for attitude estimation using gyroscope accelerometer and magnetometer.

4. COMPLEMENTARY FILTER PARAMETER TUNING

The complete complementary filter algorithm has 5 parameters in total. Using these parameters we can control the behavior of the filter, especially bias settling time (k_i), vector following speed (k_p), relevance of magnetometer over accelerometer or vice versa (k_a, k_m) and rejection of acceleration ($\boldsymbol{\sigma}$). If we set these values randomly, the filter can operate unexpectedly and can diverge. Therefore it is convenient to do parameters tuning in simulation environment. For this purpose the filter was tested in MATLAB environment.

Sensor data was generated using precise model of hexacopter. In this model very authentic models for sensor data were used (bias random walk, constant bias, white noise, misalignment, scaling factors ...). The advantage of the simulation model is the knowledge of the true attitude values which are to be estimated using complementary filter.

The parameters tuning was performed using brute-force, this means that huge 5-dimensional array of parameters was generated and the complementary filter algorithm was conducted for all sets of parameters from this array. The estimated attitude for the testing trajectory was compared with true values, and the complete sum of RMSE for whole trajectory was computed. From this RMSE database the parameters set with lowest RMSE sum was declared as the optimal. In Table 1 you can see the ranges for individual parameters, for which the test was performed.

Parameter	Start Value	Stop Value	Step
k_p	0.1	2.2	0.3
k_i	0.0001	0.0021	0.0002
k_a	0.5	3	0.5
k_m	0.5	3	0.5
$\boldsymbol{\sigma}$	0.5	1	3.5

Table 1: Parameters Intervals

The best (lowest) RMSE value was recorded for parameter set in Table 2.

Parameter	k_p	k_i	k_a	k_m	σ
Value	0.4	0.0019	0.5	3	0.5

Table 2: Best parameter set

The total RMSE sum for approximately 2 min a 40 sec long test flight was:

$$\text{RMSE} = 94.1 \tag{16}$$

The plotted Euler angles are on Figure 2.

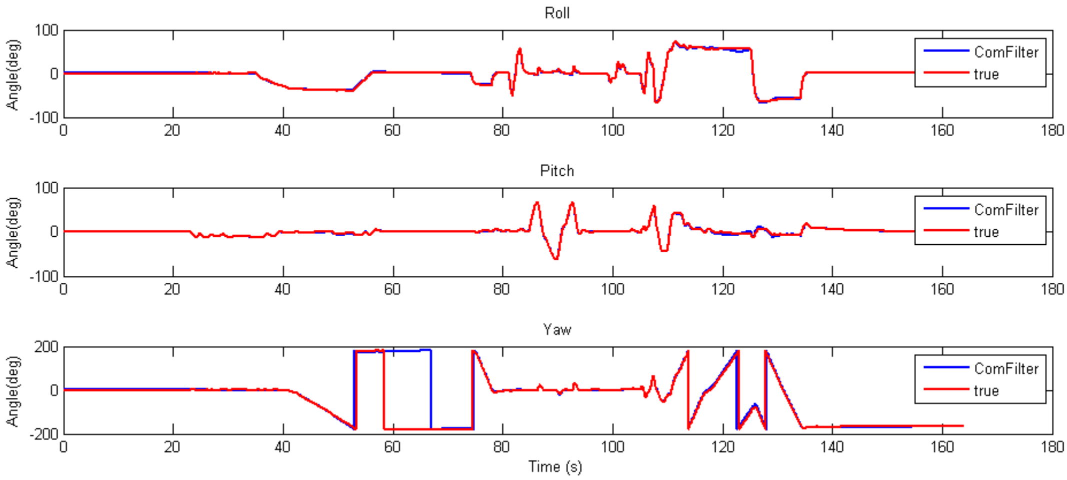

Figure 2: Euler angles for best parameter set

5. CONCLUSION

In this article the complementary filter algorithm for attitude estimation was presented. In the chapter 3 all equation of complementary filter are mentioned. The main contribution is parameter tuning using trajectory generated by precise hexacopter model. Result in Figure 2 shows sufficient performance of the filter. The future work will include implementation on a real platform with real sensors.

REFERENCES

[1] Schuster, M. D. (1993). A Survey of Attitude Representations. *The Journal of the Astronautical Sciences*, *41*(4), 439–517

[2] Marins, J. L., Bachmann, E. R., McGhee, R. B., & Zyda, M. J. (2003). An extended Kalman filter for quaternion-based orientation estimation using MARG sensors. *Proceedings 2001 IEEE/RSJ International Conference on Intelligent Robots and Systems. Expanding the Societal Role of Robotics in the the Next Millennium (Cat. No.01CH37180)*, *4*, 2003–2011.

[3] Mahony, R., Kumar, V., & Corke, P. (2012). Multirotor Aerial Vehicles - Modeling, Estimation, and Control of Quadrotor. *IEEE Robotics & Automation Magazine*, *9*(3), 20 –32.

[4] Kalman, R. E. (1960). A New Approach to Linear Filtering and Prediction Problems. *Transactions of the ASME--Journal of Basic Engineering*, Vol. 82. p. 35-45.

[5] Rogers, M. R. (2003). Applied Mathematics in Integrated Navigation Systems. *American Institute of Aeronautics and Astronautics, Inc.* Reston, Virginia. Second edition.