DISTINCTION OF INTERFERENCES IN SMOOTHED PSEUDO WIGNER-VILLE DISTRIBUTION

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Abstract: This arcticle deals with Wigner-Ville distribution, it's properties and variants, especially with smoothed pseudo Wigner-Ville distribution. Use of analytic signal is metioned and interference problem is introduced with background of interference creation. Existing methods of these so called cross-terms reduction are briefly enumerated and finally a new method for distinction of interference from autoterms is presented.

Keywords: Wigner-Ville distribution, reduced interference, signal processing, analytic signal

1 INTRODUCTION

Wigner-Ville distribution (WVD) is a time-frequency transformation deduced from Wigner distribution (WD) which is used in the field of quantum mechanics. WVD has very good time and frequency resolution which is used in many areas, from non-destructive testing across radar and sound applications to telecommunications. Disadvantage of the WVD is presence of interference terms devaluating good readability of the time-frequency transformation result.

2 WIGNER-VILLE DISTRIBUTION

Even though many authors use only the name of Wigner distribution, we should distinguish between Wigner distribution and Wigner-Ville distribution. J. Ville introduced Wigner distribution into field of signal processing by exchanging wavefunction autocorrelation by complex (analytic) signal [1]. When using analytic signal the name of Wigner-Ville distribution will be used. Application of analytic signal is essential for good final result of transformation, which was pointed out by many authors [2, 3].

2.1 ANALYTIC SIGNAL

For a real valued signal $s(t)$ we can find a complex signal $s_a(t)$:

$$s_a(t) = s(t) + jH(s(t)) \tag{1}$$

where $H(s(t))$ is a Hilbert transform of signal $s(t)$ and $s_a(t)$ is called analytic signal.

2.2 PROPERTIES OF WVD

Wigner-Ville distribution is mathematically Fourier transform of autocorrelation of the signal $s(t)$, in central form, we can write:

$$WVD(t,\omega) = \int_{-\infty}^{\infty} s(t - \frac{1}{2}\tau)s^*(t + \frac{1}{2}\tau)e^{-j\omega\tau}\,\mathrm{d}\tau \tag{2}$$

where $s^*(t)$ is a complex conjugate of the signal, which is analytic (complex) as mentioned before.

WVD has many good mathematical properties [4], especially energy conservation (integrating WVD across time-frequency plane equals energy of signal), marginal properties (integrations along axes result in spectral density and instantaneous power), translation and dilation covariance etc.

In reality the equation 2 is problematic for application because of integration from $-\infty$ to ∞. In practical use a window $h(\tau)$ is introduced however this filtration influence result only in frequency domain. Consequently a smoothed pseudo WVD was formulated to permit independent filtration (smoothing) in time and frequency [5]:

$$SPWVD(t,\omega) = \int h(\tau) \int g(\varepsilon - t) s(t - \frac{1}{2}\tau) s^*(t + \frac{1}{2}\tau) e^{-j\omega\tau} \, d\tau \tag{3}$$

2.3 Interference

Drawback of WVD is presence of interferences in case of multiple components in a signal. When two components x and y are present the result of WVD can be expressed:

$$W_{x+y}(t,\omega) = W_x(t,\omega) + W_y(t,\omega) + 2\Re\{W_{x,y}(t,\omega)\} \tag{4}$$

$$W_{x,y}(t,\omega) = \int x(t + \frac{\tau}{2}) y^*(t - \frac{\tau}{2}) e^{-j\omega\tau} \, d\tau \tag{5}$$

W_x and W_y are WVD of single components and $W_{x,y}$ represents the cross-term introduced by the quadratic nature of WVD and must be present for satisfying mentioned good mathematical properties and for having good time-frequency resolution [6].

3 EXISTING METHODS OF INTERFERENCE REDUCTION

In the problematic of interference reduction many methods were proposed in last two decades: simple smoothing kernel optimization [7], non-linear filtering [8], S-method [9], use of fractional Fourier [10], empirical mode decomposition (EMD) [11] and even other methods combining WVD with other distributions or signal processing techniques [12].

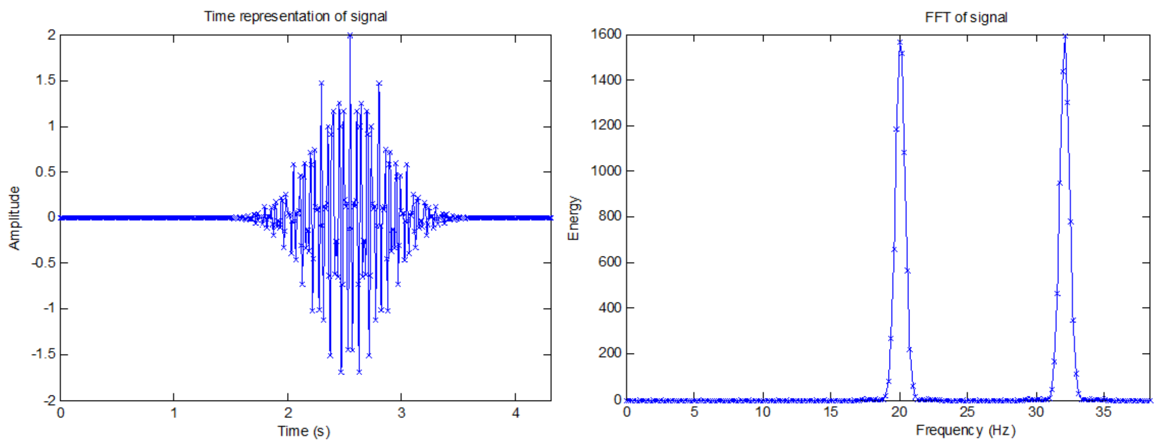

Figure 1: Simulated signal and its Fast Fourier transform.

4 DISTINCTION OF INTERFERENCE IN SPWVD

New proposed method to distinguish interferences from auto terms can be presented on a signal with two monocomponent gauss time-frequency atoms with different frequency and same time of appearance. Time and frequency representation of this simulated signal is on figure 1.

Result of non-filtered SPWVD is on left of figure 2. We can clearly see three events (two autoterms and one interference in center of them). On same figure time smoothing effect on interference reduction is visible - growing window size reduces cross-term.

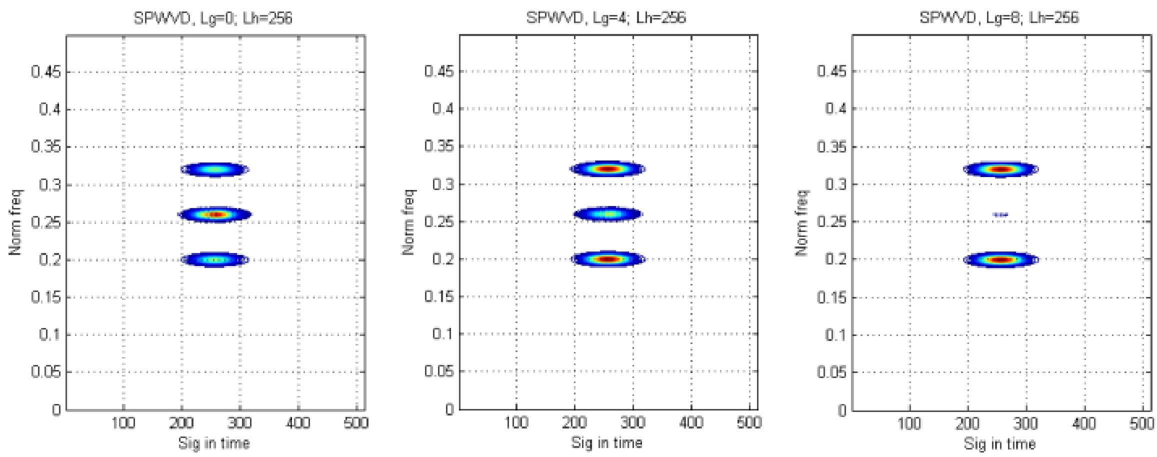

Figure 2: WVD of the signal - erasing of interference by time smoothing. Time smoothing windows (except central point) from left: 0, 4 and 8 samples.

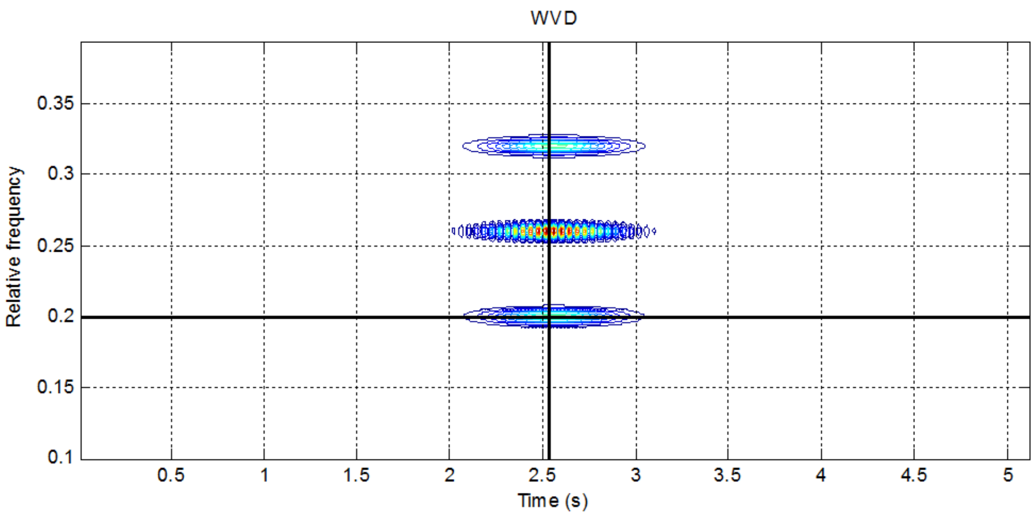

Figure 3: WVD of the signal.

If we focus on constant time or frequency of WVD (horizontal and vertical lines on figure 3), the variation of amplitudes created by different time smoothing windows can be plotted in a graf (figure 4).

With wider window the autoterm amplitude is falling down and becomes less focused (left part of figure 4), however the amplitude of interference falls much more fast (right part of figure 4).

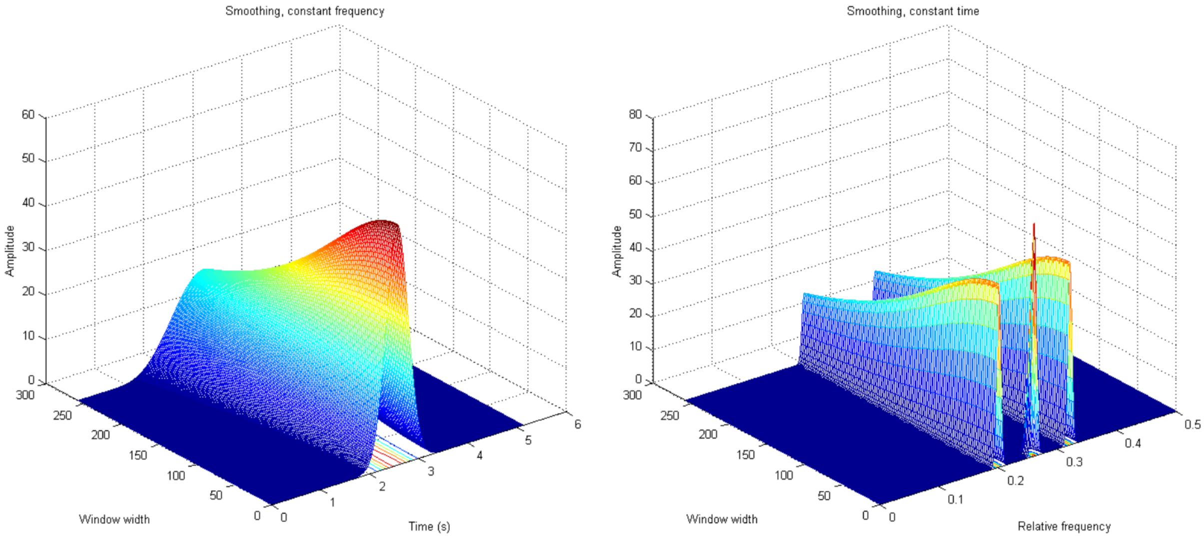

Figure 4: Influence of smoothing on resolution quality and interference amplitude (left: influence of different window size on horizontal line from figure 3, right: same on vertical line from same figure).

This basic idea is used in optimal kernel searches (finding optimal kernel for optimal ratio interference reduction - autoterm clarity and amplitude). Our idea is to use different time-frequency smoothing windows and from amplitude decrease classify time-frequency point as autoterm, interference or summed interference with autoterm.

Example result of classification of five Gaussian atoms with noise is on figure 5.

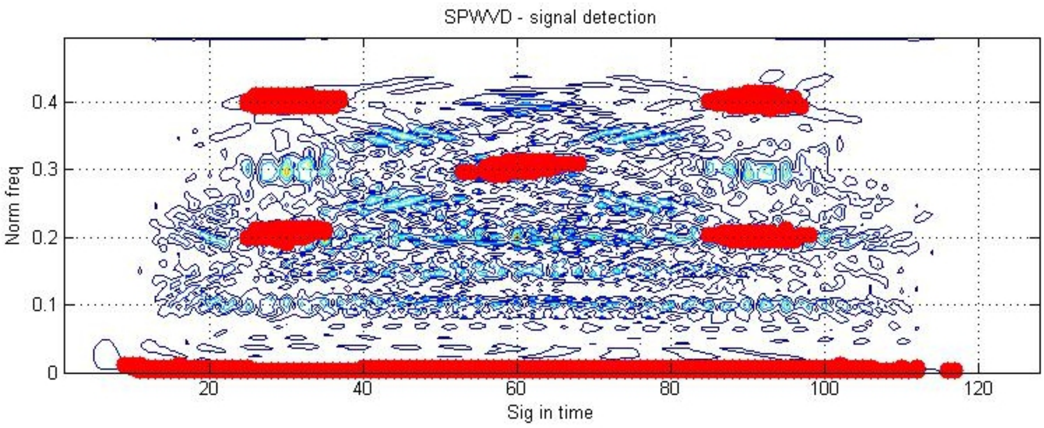

Figure 5: Classification of five Gaussian atoms (highlighted in red) in noised signal.

5 CONCLUSION

Basic theory of WVD was introduced with mention of analytic signal importance. Interference problematic was described with brief list of existing methods of interference reduction.

A new method based on interference attenuation by different smoothing window in SPWVD was explained on case of two Gaussians atoms separated in frequency. Results were presented on five component noisy signal.

In future work algorithm would be tested more deeply, interference amplitude estimation and attenuation will be implemented and it's results will be compared with other existing methods.

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