

HYPERBOLIC BROWSER IN WEB ENVIRONMENT

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Abstract: This paper deals with the graphic portrayal of large hierarchies, exploring possibilities of tree graphs visualization. The aim is to provide a suitable mathematical model for implementation of a hyperbolic browser in web environment. The browser shall exploit the potential of non-euclidean geometry and shall project trees onto hyperbolic plane.

Keywords: EEICT, hyperbolic, browser, tree, poincaré

1 INTRODUCTION

Since massive expansion of computer science as a tool to process nearly any kind of information, there has always been a need to visualize the information in the nicest and friendliest way possible. This paper uses the idea of the Hyperbolic Browser, published and patented in 1995 by J. Lamping and R. Rao from Xerox Palo Alto Research Center, and describes a suitable mathematical model for graphic portrayal of large tree structures.

2 TREE

Tree is a hierarchical data structure, accessed starting at the root node. Each node is either a leaf or an internal node. An internal node has one or more child nodes and is called the parent of its child nodes. All children of the same node are siblings. It is a connected, undirected, acyclic graph and it is rooted and ordered unless otherwise specified[1].

3 HYPERBOLIC PLANE AND GEOMETRY

In mathematics, hyperbolic geometry (also called Lobachevskian geometry) is a non-Euclidean geometry, because the parallel postulate of Euclidean geometry is not fulfilled[2]. The parallel postulate of Euclidean geometry says that, in two dimensional space, for any given line l and point P not on l , there is one and only one line k through the point P which has no intersection with line l , i.e., lines l and k are parallel. In hyperbolic geometry there are at least two distinct lines through P which do not intersect l , so the parallel postulate does not hold.

There are four different commonly used models, that define a real hyperbolic space and satisfy the axioms of a hyperbolic geometry.

In a two-dimensional Poincaré disk model, a point is represented by a pair of real numbers (x_1, x_2) where $x_1^2 + x_2^2 < 1$, i.e., point of an open disk in the complex plane. The distance between two such points is given by

$$d(x, X) = a \cosh^{-1} \left[\frac{1 - x_1 X_1 - x_2 X_2}{\sqrt{1 - x_1^2 - x_2^2} \sqrt{1 - X_1^2 - X_2^2}} \right] [3].$$

4 LAYOUT

Assume that each tree node has a given wedge of the circle used in the hyperbolic browser canvas. The node itself is placed at the vertex of its wedge. The angle of the wedge is divided by the number of children, so that every child has its own *subwedge*. Children are placed along the arc in the node's wedge at an equal distance from their parent. Because the lines in hyperbolic plane diverge as mentioned above, the subwedge of each child can span the same angle of the wedge as their parent owns, and yet no descendant's subwedge overlaps[4].

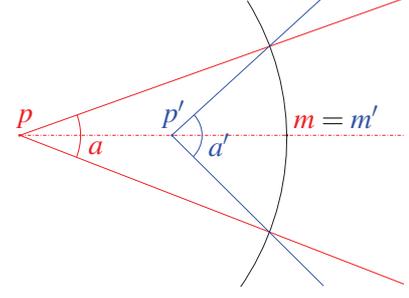


Figure 1: Wedge and Subwedge

The layout is computed recursively for each node, starting from the root and going through all its children using post-order depth first search traversal. The distance from the node to its child is computed by

$$d = \sqrt{\left(\frac{(1-s^2)\sin(a)}{2s}\right)^2 + 1} - \frac{(1-s^2)\sin(a)}{2s}, \quad (1)$$

where a is an angle between midline and edge of the subwedge, and s is the desired distance between the child and the edge of its subwedge. d is the computed distance. All values are scalars; d and s are represented as a hyperbolic tangent of the distance in hyperbolic plane[4].

Given the subwedge for the child and the distance, it is possible to compute the child's wedge inside the subwedge, as seen in figure 1. Given the subwedge's vertex p , midline endpoint m and angle of the subwedge a , computed wedge of the child will be calculated using the transformation

$$z_t = Trans(z, \langle P, \theta \rangle) = \frac{\theta z + P}{1 + \overline{P}\theta z}, \quad (2)$$

where the new position z_t is the old position z rotated by angle θ and moved to P .

Vertex calculation:

$$p' = Trans(dm, \langle p, 1 \rangle) = \frac{dm + p}{1 + \overline{dm}p}. \quad (3)$$

Middle endpoint:

$$m' = Trans(Trans(m, \langle p, 1 \rangle), \langle -p', 1 \rangle) \quad (4)$$

$$= Trans\left(\frac{m+p}{1+\overline{pm}}, \langle -p', 1 \rangle\right) \quad (5)$$

$$= \frac{\frac{m+p}{1+\overline{pm}} - p'}{1 + \overline{-p'}\frac{m+p}{1+\overline{pm}}}. \quad (6)$$

Angle of the wedge:

$$a' = \Im(\log(Trans(e^{ia}, \langle -d, 1 \rangle))) \quad (7)$$

$$= \Im(\log(Trans((\cos a + i \sin a), \langle -d, 1 \rangle))) \quad (8)$$

$$= \Im\left(\log\left(\frac{(\cos a + i \sin a) - d}{1 + \overline{-d}(\cos a + i \sin a)}\right)\right), \quad (9)$$

where $\Im \log(\dots)$ returns an angle of the wedge converted from the complex number returned from the transformation. The number e^{ia} is a complex number in trigonometric form $re^{i\alpha} = r(\cos \alpha + i \sin \alpha)$, and for this case $r = 1$ simply $e^{ia} = \cos a + i \sin a$.

5 PROJECTION

Once all nodes are laid out onto the hyperbolic plane, every node stores its own position, which is permanent, i.e., it is never changed in the future unless the graph structure is modified.

A mathematical theory of the Poincaré model is now used for mapping the nodes from the hyperbolic plane to the unit disk. To have the root in the middle of the canvas, while further neighborhood is fading off towards the edge of the unit disk, the initial view projection $\langle P, \theta \rangle$ shall be set to $\langle 0, 1 \rangle$.

Changing the projection is quite simple. An inverse mapping of the selected point on the unit disk is computed to get the corresponding point in the hyperbolic plane. Given the current transformation $\langle P, \theta \rangle$, the inverse transformation is computed by $P' = -\bar{\theta}P$ and $\theta' = \bar{\theta}$. The point in the hyperbolic plane is then calculated $z_t = Trans(z, \langle P', \theta' \rangle)$, where z_t is the point on the hyperbolic plane and z is a point selected in the unit disk. The new graph's visualization is achieved by the composition of the current view projection and the new view projection. The composition $\langle P, \theta \rangle$ of the transformations $\langle P_1, \theta_1 \rangle$ and $\langle P_2, \theta_2 \rangle$ is given by $P = \frac{\theta_2 P_1 + P_2}{\theta_2 P_1 P_2 + 1}$ and $\theta = \frac{\theta_1 \theta_2 + \theta_1 \bar{P}_1 P_2}{\theta_2 P_1 P_2 + 1}$.

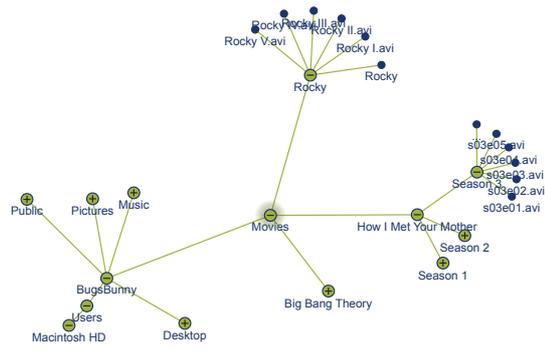


Figure 2: Screenshot of the Browser

6 CONCLUSIONS

This paper presents both efficient and effective way how to visualize and browse large tree structures in web environment. The hyperbolic browser is implemented in JavaScript and supported by all modern browsers. It brings a very uncommon and yet highly comfortable browsing experience.

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