SIGNAL FILTERING USING RECTANGULAR PULSE IN SPECTRAL DOMAIN

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Abstract: This paper describes the characteristics of spectral filtering when using a rectangular pulse. It discusses conditions for an ideal spectral filtering. It shows when parasitic sounds appear and defines conditions to suppress them.

Keywords: FFT, rectangular pulse, spectral filtering

1 INTRODUCTION

Audio signals are usually filtered by the IIR or the FIR filter, where the order of the filters determines the steepness of them. Thus, if we use cutting edge filters, we have to use a high order filter [1].

Filtering in spectral domain looks like an easy way to emulate an ideal steep filter. It does this by reducing unwanted frequencies in the spectral domain. But parasitic frequencies appear when considering the real application of filtering in spectral domain [4]. The purpose of this article is to describe these parasitic sounds and to suggest methods to suppress them.

2 DESCRIPTION OF FILTERING

Filtering in spectral domain means the suppression (or setting it to zero) of the desired spectral components of a discrete signal $x(n)$. This is the equivalent to the multiplication of the frequency spectrum $X(k)$ with a window $W(k)$, where

$$W(k) = \begin{cases} 1 & \text{for the components we want to keep,} \\ 0 & \text{for the components we want to reduce.} \end{cases}$$

We get a final signal $y(n)$ by the IDFT of the modified spectrum

$$y(n) = \mathrm{IDFT}\{X(k)W(k)\}. \tag{1}$$

3 IDEAL FILTER

In some specific conditions the parasitic frequencies do not appear.

If the signal $x(n)$ contains only the frequency components of f_d, where d takes the values 1 through $Ns/2$ to which

$$f_d = \frac{N_s f_s}{2b}, \text{where } b \in \mathrm{Z}, \tag{2}$$

$$f_d \leq \frac{f_s}{2}, \tag{3}$$

For example, let us have the signal $x_s(n)$, which consists of three harmonic frequencies with sine $f_1 = 312.5$ Hz, $f_2 = 1250$ Hz, $f_3 = 1812.5$ Hz. To minimize the impact of STFT (short-time Fast Fourier transform) a sufficiently large number of the $N_{with} = 2^{15}$ signal samples $x_s(n)$ must be used. This signal is split to the segment $x_{si}(n)$ size of $N = 256$ to calculate the STFT. The sampling frequency $f_s = 4$ kHz.

$$x_s(n) = \frac{1}{3}\sin(2\pi n \frac{f_1}{f_s}) + \frac{1}{3}\sin(2\pi n \frac{f_2}{f_s}) + \frac{1}{3}\sin(2\pi n \frac{f_3}{f_s}) \tag{4}$$

Spectrum of signal $X_s(k)$ is shown in Fig. 1.

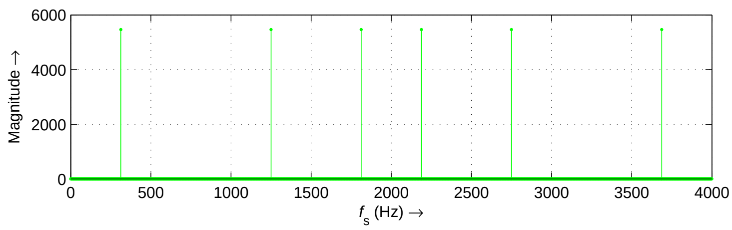

Figure 1: Spectrum of signal $X_s(k)$, $N_{\text{STFT}} = 2^{15}$,$f_s = 4$ kHz

We want to design a rectangular window $W(k)$ that suppresses f_1 and f_3 and keep f_2, so the passband is selected from $f_{p1} = 700$Hz to $f_{p2} = 1500$Hz.

At first, it is necessary to calculate resolution δf of the spectrum for STFT.

$$\delta f = \frac{f_s}{N} = 15.325\text{Hz}, \tag{5}$$

and find the nearest multiple of δf for f_{p1} a f_{p2}.

$f_{p1} = 687.5$; is the 44 spectral component
$f_{p2} = 1500$; is the 96 spectral component,

Because the $x_{si}(n)$ is real, the window has to be symmetrical around the center ($f_s/2$) [3]), that

$$W(k) = \begin{cases} 1 & \text{if } k \in \langle 44, 96\rangle \cup \langle 160, 212\rangle \\ 0 & \text{if } k \in \langle 0, 43\rangle \cup \langle 97, 159\rangle \cup \langle 213, 255\rangle. \end{cases} \tag{6}$$

The results design the rectangular pulse $W(k)$ displayed in Fig. 2.

We split the signal $x_s(n)$ on the segment $x_{si}(n)$ size of 256, where $i \in \langle 0, 127\rangle$. Each segment is converted into the frequency domain and multiplied with the window $W(k)$. After we calculate inverse DFT and assign to the output sequence of $y(n)$(without attribution of overlap).

In Fig. 3 it can be seen the spectrum $Y(k)$ after filtering. It is clear that the frequencies f_1 and f_3 are ideally suppressed without any parasitic effects.

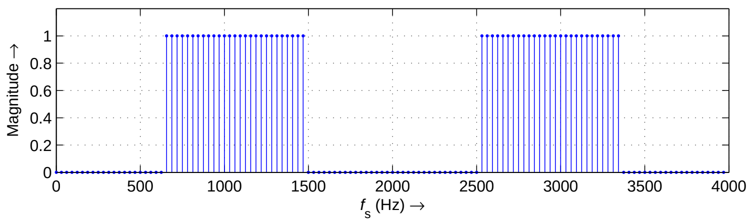

Figure 2: Window $W(k)$

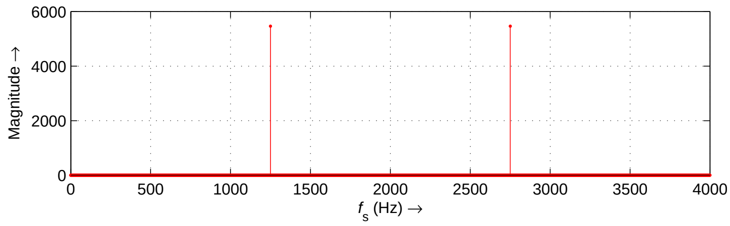

Figure 3: Signal spectrum $X_s(k)$ after filtering, $N_{\text{STFT}} = 2^{15}$,$f_s = 4$ kHz

4 REAL FILTER

If we have a discrete signal x_z, it does not comply with the conditions of Equation (2) and (3). When we apply filtering in the spectral domain on this signal the new (parasitic) frequencies appear. In the example signal (1) we change the frequencies f_1, f_2, f_3 thus the period of these frequencies are not multiply with N. The selected frequencies are $f_1 = 300$, $f_2 = 1300$, $f_3 = 1600$. All other parameters including the window $W(k)$ are same. The spectrum of this signal is shown in Fig. 4. Because any period of frequencies $f1, f2$ and $f3$ are not multiplies of $N_s = 2^{15}$ the energy of spectral components is slightly spread.

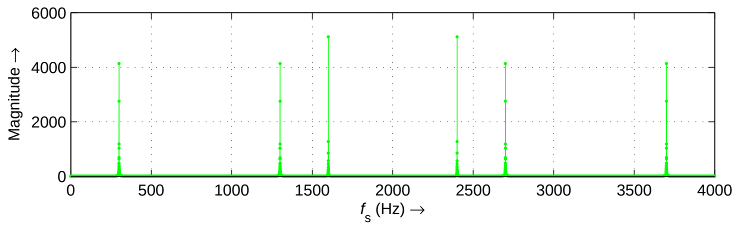

Figure 4: Spectrum of signal $X_z(k)$ $N_{\text{STFT}} = 2^{15}$,$f_s = 4$ kHz

The spectrum of signal after the filtering is in Fig. 5. It can be seen as components f_1 and f_2 are suppressed, but new(parasitic) frequencies appear near 650 Hz and 1500 Hz.

To describe these characteristics we use properties, the multiplication of two signals in spectral domain is equivalent to a circular convolution of these signals in time domain [2],

$$y(n) = \text{IDFT}\{X_i(k)W(k)\} \equiv x_i(n) \otimes w(n). \tag{7}$$

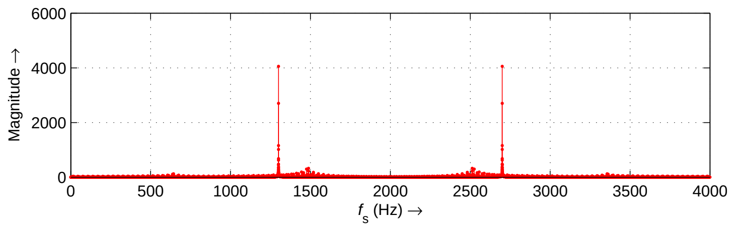

Figure 5: Spectrum of signal $X_z(k)$ after filtering, $N_{\mathrm{STFT}} = 2^{15}$,$f_s = 4$ kHz

As we see, the output signal $y(n)$ is affected by the transforming characteristic of the rectangular pulse $w(n)$. And it can be expressed as

$$w(n) = \mathrm{IDFT}\left[\mathrm{W}(k)\right]. \tag{8}$$

To derive the formula for the inverse transform of the rectangular pulse we use a formula for forward transform of the rectangular pulse $r_w(n)$ to spectral domain $R_w(k)$, that is [4]

$$R_w(k) = \sum_{n=-\frac{M-1}{2}}^{\frac{M-1}{2}} e^{-j\omega n} = \frac{\sin(M\frac{\omega}{2})}{\sin(\frac{\omega}{2})}, \tag{9}$$

where M is the number of samples which rectangular pulse $r_w(n)$ was sampling. Then relation between DFT and IDFT is [2]

$$\mathrm{IDFT}[W(k)] = \frac{1}{N}\mathrm{DFT}[W(k)^*] = w(n) \tag{10}$$

where $W(k)^*$ is complex conjugate.

After substitution into equation (9) we get

$$w(n) = \frac{1}{N}\sum_{k=0}^{N} e^{j\omega k} = \frac{\sin(M\frac{\omega}{2})}{N\sin(\frac{\omega}{2})}. \tag{11}$$

Because rectangular pulse can have shift δ_k from beginning we apply sentence of linearity [4] about linear shifting

$$w(n) = e^{j\omega\delta_k}\frac{1}{N}\sum_{k=\delta_k}^{\delta_k+M} e^{j\omega k} = \frac{2\cos(\omega\delta_k)\sin(M\frac{\omega}{2})}{N\sin(\frac{\omega}{2})}, \tag{12}$$

5 CONDITIONS OF FILTERING

In the previous chapter 4 the characteristic of transform rectangular window was derived. Example of this function is shown in Fig.6. This function is periodic and because the side lobs have theoretical infinitive length, the lobs overlap will occur. If we want to discover the relationship between effects of overlap to result signal we have to found out the relationship between local maximum at beginning ($n = 0$) and in the half of period ($n = N/2$ where the overlap is occur). For simplification, we expect that M is odd (thus $\sin(M\pi/2) = \pm 1$), and also that the offset is zero.

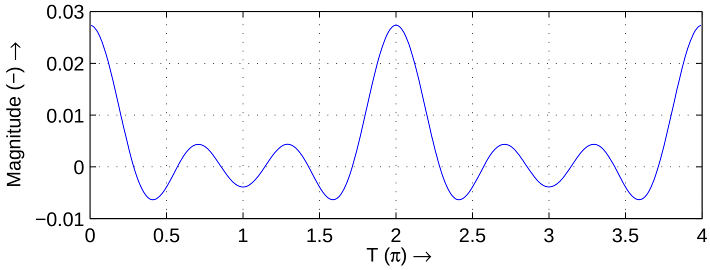

Figure 6: Two periods of signal $w(n)$, $N = 256, M = 7, \delta_k = 0$.

$$w(0) = \lim_{n \to 0} \frac{\sin(M\pi n)}{N\sin(\pi n)} = \frac{M}{N}, \tag{13}$$

(At time zero the equation (12) is not defined that the limit function is used in (13).)

$$w(N/2) = \frac{1}{N}\frac{\sin(M\frac{\pi}{2})}{\sin(\frac{\pi}{2})} = \frac{1}{N}, \tag{14}$$

The maximum difference between local maxims of signal $w(n)$ subscribe the suppression of the parasitic frequencies. That the conditions for attenuation A_l of parasitic frequencies is

$$A_l = 20\log_{10}\left(\frac{\frac{1}{N}}{\frac{M}{N}}\right) = 20\log_{10}\left(\frac{1}{M}\right). \tag{15}$$

Therefore if the attenuation of parasitic frequencies under 40 dB is required the rectangular window $W(k)$ with at least 100 nonzero components must be designed.

6 CONCLUSION

This paper describes the properties of filtering using rectangular pulse in spectral domain. The IFFT of discrete rectangular pulse is not the function *sinc*, as we can expect, but it is more complex equation(12). Because the segment in time domain is limited the overlap is occur and it shows up like a new parasitic frequencies. The attenuation of parasitic frequencies is indirectly depending on number of components obtained in rectangular window (15).

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