PLANCK AND ROSSELAND MEANS OF ABSORPTION COEFFICIENTS IN AIR

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ABSTRACT

Mean absorption coefficients (Planck, Rosseland, and arithmetic means) were calculated for thermal air plasma as a function of plasma temperature in the range of 1 000 - 35 000 K at the plasma pressure of 10^5 Pa in 10 specific frequency intervals of the frequency interval 10^{13}-10^{16} s^{-1}.

1. INTRODUCTION

The propagation of thermal radiation is an integral part of the hydrodynamic description of a plasma motion. In description of radiation transfer, the mathematical modeling is of great importance. Due to the non-linearity of equations describing the radiation field and strong dependence of input parameters on the radiation frequency, various approximate methods are used. One of them is the method of spherical harmonics (P_N – approximation). Mainly the lowest order P_N solution corresponding to $N = 1$ (P_1) is used (so called diffusion approximation). The diffusion approximation describes in good accuracy the radiation field in many problems of radiation hydrodynamics. The higher order P_N – approximations make the calculation algorithms very complicated. The diffusion approximation is relatively simple, it enables to find the radiative heat flux and radiative energy density by solving a system of equations that do not depend on angular variables Ω.

2. EQUATION OF RADIATION TRANSFER

The equation of radiation transfer has the following form

$$\vec{\Omega} \cdot \text{grad } I_\nu = k_\nu (I_\nu^{eq} - I_\nu) \tag{1}$$

where $\vec{\Omega}$ is the unit vector defining the radiation direction, k_ν is the absorption coefficient per unit length at frequency ν, I_ν is the monochromatic intensity of radiation, and I_ν^{eq} is the specific blackbody radiation intensity which is given by Planck formula

$$I_\nu^{eq} = B(T) = \frac{2h\nu^3}{c^2} \cdot \frac{1}{e^{\frac{h\nu}{kT}} - 1} \tag{2}$$

here, h is the Planck constant, ν is the frequency of radiation, c denotes the velocity of light, k is the Boltzmann constant, and T is the temperature.

In P_1 approximation we suppose that the angular dependence of the specific intensity can be represented by the first two terms in a spherical harmonic expansion

$$I_\nu(r,\nu,\Omega) = \varphi_1(r,\nu) + 3\vec{\varphi}_2(r,\nu)\cdot\vec{\Omega} \tag{3}$$

where φ_1 and $\vec{\varphi}_2$ correspond to the density of the radiation field multiplied by velocity of light c, and to the radiation flux.

The spectral density of the radiation field is the amount of radiation energy in $d\nu$ centered at ν included with volume element

$$U_\nu(r,\nu) = \frac{1}{c}\int_0^{4\pi} I_\nu(r,\nu,\Omega)\,d\Omega = \frac{1}{c}\int_0^{4\pi}\left[\varphi_1(r,\nu) + 3\vec{\varphi}_2(r,\nu)\cdot\vec{\Omega}\right]d\Omega = \frac{4\pi}{c}\varphi_1(r,\nu) \tag{4}$$

since

$$\frac{1}{c}\int_0^{4\pi} 3\vec{\varphi}_2(r,\nu)\cdot\vec{\Omega}\,d\Omega = 0.$$

Similarly, for radiative heat flux

$$\vec{W}_\nu(r,\nu) = \int_0^{4\pi} I_\nu(r,\nu,\Omega)\vec{\Omega}\,d\Omega = \int_0^{4\pi}\left[\varphi_1(r,\nu) + 3\vec{\varphi}_2(r,\nu)\cdot\vec{\Omega}\right]\vec{\Omega}\,d\Omega = 4\pi\vec{\varphi}_2(r,\nu) \tag{5}$$

Therefore, we may rewrite equation (3) in terms of radiation field density and radiation flux

$$I_\nu(r,\nu,\Omega) = \frac{c}{4\pi}U_\nu(r,\nu) + \frac{3}{4\pi}\vec{W}_\nu(r,\nu)\cdot\vec{\Omega} \tag{6}$$

Integrating the equation of radiation transfer (6) over all solid angles we obtain

$$\operatorname{div}\vec{W}_\nu(r,\nu) + k_\nu c U_\nu(r,\nu) = k_\nu c U_\nu^{eq}(r,\nu) \tag{7}$$

where $U_\nu^{eq}(r,\nu)$ is the spectral density of equilibrium radiation

$$U_\nu^{eq}(r,\nu) = \frac{8\pi h\nu^3}{c^3}\cdot\frac{1}{e^{\frac{h\nu}{kT}} - 1} \tag{8}$$

Similarly, integrating the equation of transfer (6) multiplied by $\vec{\Omega}$ we obtain

$$\frac{1}{3}c\cdot\operatorname{grad}U_\nu + k_\nu W_\nu = 0 \tag{9}$$

The diffusion approximation is valid under assumption that the specific intensity of radiation is almost isotropic.

3. ABSORPTION PROPERTIES OF AIR

If I_ν is the monochromatic intensity of radiation passing through an absorbing medium of thickness dx, then k_ν, the absorption coefficient per unit length at frequency ν, is given by

$$d\,I_\nu = -k_\nu I_\nu dx \tag{10}$$

Quantity k_ν is generally expressed in cm^{-1}. It is in general a function of the wavelength, the gas properties at the frequency ν, and the direction in which the radiation propagates.

From the known frequency variations of absorption coefficients in plasma temperature range $T = (1\,000, 35\,000)$ K (with temperature step 1 000 K) the mean values of absorption coefficient at selected frequency intervals can be found.

In Fig. 1 calculated absorption coefficients of air plasma at the pressure 10^5 Pa for two temperatures 1 000 K and 10 000 K are shown. The main part of line radiation is observed for frequencies just about to 4.5×10^{15} s^{-1}. To obtain more accurate mean values of absorption coefficients, the whole frequency interval $(0.01 - 10) \times 10^{15}$ s^{-1} has been divided into 10 parts given in Table 1.
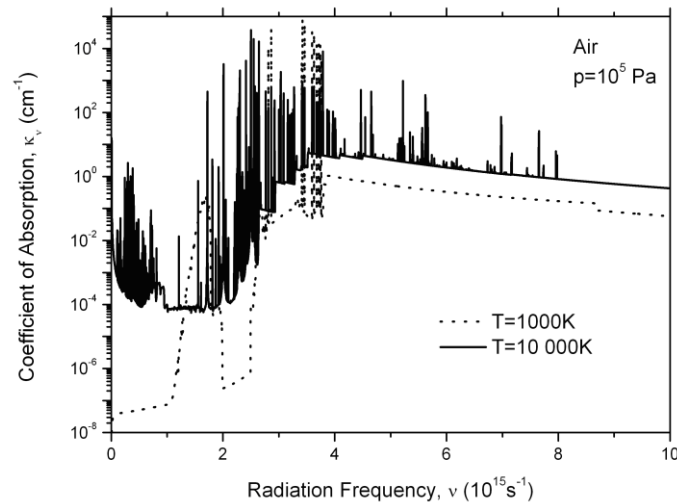
Fig. 1. Absorption coefficient in air thermal plasma as a function of frequency for temperatures 1 000 K and 10 000 K (calculated using software [5]).

Tab. 1. Limits of specific frequency intervals.

interval	1	2	3	4	5	6	7	8	9	10
ν (10^{15}s^{-1})	0.03 - 0.057	0.057- 0.296	0.296- 0.386	0.386- 0.746	0.746- 0.986	0.986- 1.71	1.71- 2.098	2.098- 2.64	2.64- 2.997	2.997- 4.49

4. THE MULTIGROUP METHOD – PLANCK AND ROSSELAND MEANS

For accurate description of real spectrum it is necessary to calculate the absorption coefficients with the frequency step less than the molecular and atomic lines width. All averaged quantities can be calculated by spectral integrations for specific spectral intervals.

Three types of average absorption coefficients have been calculated for temperatures $T \in (1\,000, 35\,000)$ K at the air plasma pressure $p = 10^5$ Pa in frequency intervals given in Tab. 1.

Planck mean is appropriate in the case of optically thin, emission dominated system. It is determined by

$$\bar{k}_P = \frac{\int\limits_0^\infty k(v)B(v)dv}{\int\limits_0^\infty B(v)dv} \qquad (11)$$

where $k(v)$ is the spectral absorption coefficient, and $B(v)$ is the Planck equilibrium radiation intensity (2)

Rosseland mean is also called mean free path of radiation is appropriate when the system approaches equilibrium (almost all radiation is reabsorbed). It is given by

$$\overline{k^{-1}} = \frac{\int\limits_0^\infty k^{-1}(v)\dfrac{dB(v)}{dT}dv}{\int\limits_0^\infty \dfrac{dB(v)}{dT}dv} \qquad (12)$$

where the temperature derivative of Planck function is

$$\frac{dB}{dT} = \frac{2h^2v^4}{kT^2c^2} \cdot \frac{e^{\frac{hv}{kT}}}{\left(e^{\frac{hv}{kT}} - 1\right)^2} \qquad (13)$$

Arithmetic mean

$$\bar{k} = \frac{\sum\limits_{i=1}^n k_i}{n} \qquad (14)$$

where k_i is the absorption coefficient corresponding to given frequency, and n is the number of discrete frequency values in specific frequency interval.

5. RESULTS AND DISCUSSION

Mean absorption coefficients of air plasma (Planck, Rosseland and arithmetic means) have been calculated for 10 specific frequency intervals at the pressure $p=10^5$ Pa. Comparison of three different mean values for three selected frequency intervals is given in Fig. 2.

6. CONCLUSION

The use of Rosseland and Planck mean absorption coefficients is only strictly appropriate in limiting circumstances. In the multigroup method, the splitting of the whole frequency interval has to be made according to absorption coefficients frequency dependence. For frequency groups with low values of absorption coefficients, the use of Planck mean is appropriate; for groups with high values of absorption coefficients, the Rosseland mean is more suitable. In our case, for groups 1-9 the Planck mean will be used, in the group 10 the Rosseland mean will give better results. The arithmetic mean is mentioned only for comparison.
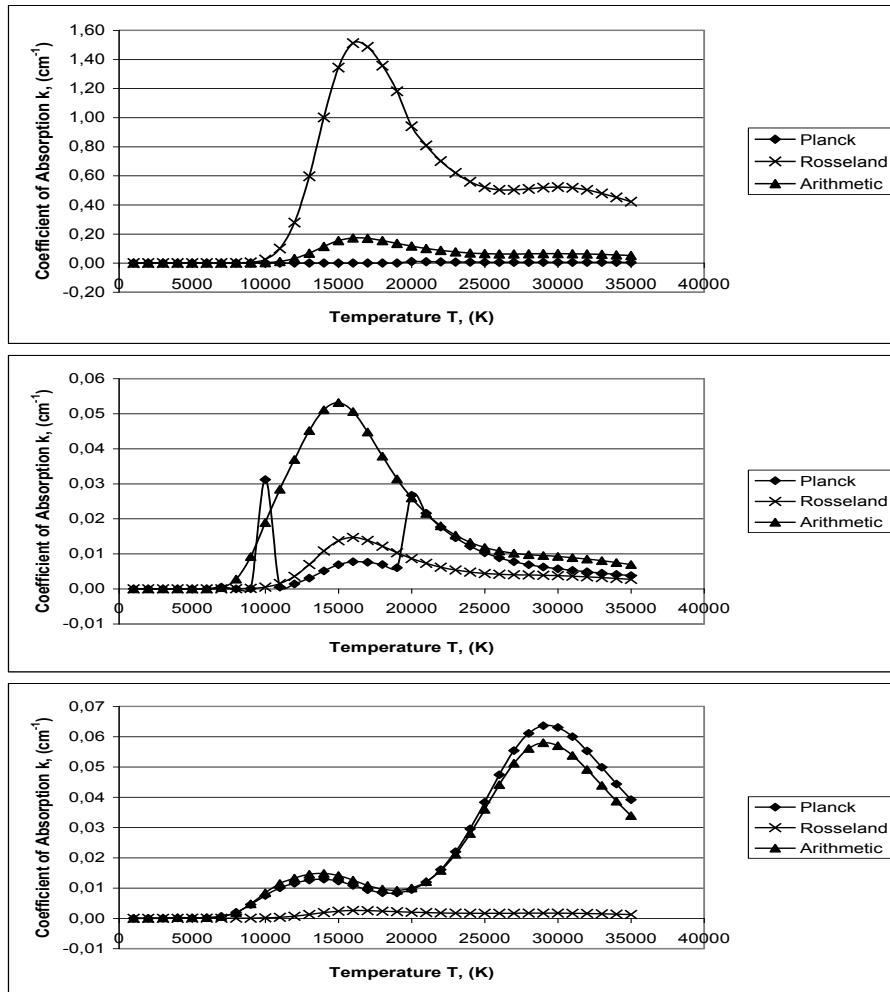
Fig. 2. Mean absorption coefficients of air (Planck, Rosseland and arithmetic means) in the frequency intervals: $(0.037-0.057) \times 10^{15}$ s^{-1}, $(0.057-0.296) \times 10^{15}$ s^{-1} and $(0.386-0.746) \times 10^{15}$ s^{-1}.

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