USE OF SCATTERING MATRIX IN BODY TISSUE INVESTIGATION

Hamed Mohamed Abubaker
Doctoral Degree Programme (1), FEEC BUT
E-mail: xabuba00@stud.feec.vutbr.cz

Supervised by: Pavel Tománek
E-mail: tomanek@feec.vutbr.cz

ABSTRACT

The living body tissues consist of cells which dimensions are bigger than a wavelength of visible light. Therefore a Mie scattering of transmitted or backscaterred light occurs and different polarization states arise. We have characterized the bio-material with Mueller matrix optical polarimetry. The images and matrix elements with Mueller calculation provides a comprehensive information of samples due to which is possible to combine all the necessary parameters for describing beam of light into a single image. Therefore the light beam is simply described by four-parameter Stokes vector and determined by measuring a flux transmitted through a set of polarization optics, polarization generating optics provide linear and circular polarized light to sample and polarization analyzing optics collect polarized output light from sample to detecting devices.

1. INTRODUCTION

Light scattering is widely used in biological research to determine particle numbers, particle sizes, axial ratios, size distributions, particle mobilities, and indices of refraction [1].

When an obstacle is illuminated by an EM wave, electric charges in the obstacle are set into oscillatory motion by the electric field of the incident wave. The accelerated charges reradiate EM energy in all directions. This secondary radiation is called the radiation scattered by obstacle. If the frequency of scattered light is the same as that of incident light, then the scattering event is caller as elastic scattering. If the incident EM energy is transformed into thermal energy, then the process is called as absorption.

The biological materials and other random inhomogenous media render imaging difficult due to the random multiple scattering of light. Inhomogeneities in the media cause scattering which may alter the direction of propagation, polarization and phase of the light. The propagation of light through such media may be analyzed either by means of the wave or the photon theory, respectively. Photons travel in straight line paths until they encounter an inhomogeneity, where they are scattered in random directions. Most of studies measure only the small-angle differential scattered light intensity even though much more additional information is contained in the polarization states of the differentially scattered light.

2. METHOD

In the turbid medium made up of a random aggregate of scatterers, the photon undergo repeated scattering. The turbid medium is characterized by the scattering mean free path l_s, which is mean distance the photons travel before getting scattered, and the transport mean free path l^* which is the mean distance photons travel before the direction of propagation is randomized. Since it is quite possible that photons are forward scattered (and continue to travel in the same direction), $l^* > l_s$. The transport mean free path depends upon the number density of scatterers, refractive index contrast between the medium and the scatterers, and the anisotropy factor, i.e. a factor quantifying the directional distribution of scattering. Typical values of l^* for IR light in tissue are 1-2 mm. The light emerging from turbid medium consists of three components [2], the ballistic, the diffusive and the snake photons. These differ in their paths through medium, and consequently in their imaging properties. The un-scattered of forward scattered photons travel un-deviated and emerge the first, having traveled the shortest distance through the medium. These preserve the characteristics of the incident light, namely direction of propagation, polarization and are hence best for imaging. However, they are few in number, their intensity falling off as $I(z) = I_0 \exp(-z/l^*)$, where I_0 is the input intensity, and z is the distance traveled.

2.1. SCATTERING MATRIX

The formal theory of elastic light scattering deals with Maxwell's equations, boundary conditions, and idealized physical models for the scatterer, matrix algebra, which manipulates the interaction matrices, and light vectors, which describe the optical system, ignoring the exact mechanism causing the scattering. The fundamental problem of deriving structural features of the scatterer from scattering information is presently unsolvable for most biophysical cases. However, the matrix algebra can be used to catalog the scattering signals, which are related to the matrix elements involved in the scattering process.

A detailed discussion of the general scattering matrix is given in van de Hulst [3]. We will review the role of Rayleigh and Mie particles and their relationships to biological particles, and then give an example of the matrix multiplication implied by a particular arrangement of the optical components to show how matrix elements are measured. We will then discuss some matrix element signals we measured to show their significance to biological systems. A scatterer changes the state of the incoming polarized light by mixing the initial polarization states of the incident electric field vectors $E_{\parallel 0}$, and $E_{\perp 0}$. Here $E_{\parallel 0}$, and $E_{\perp 0}$ are the initial components parallel and perpendicular to the scattering plane, respectively. The new parallel and perpendicular electric field components $E_{\parallel}$ and $E_{\perp}$ arise through an interaction represented by mixing coefficients A_i.

$$\begin{aligned} E_{\parallel} &= A_2 E_{\parallel 0} + A_3 E_{\perp 0} \\ E_{\perp} &= A_4 E_{\parallel 0} + A_1 E_{\perp 0} \end{aligned} \quad \text{or} \quad \begin{vmatrix} E_{\parallel} \\ E_{\perp} \end{vmatrix} = \begin{bmatrix} A_2 & A_3 \\ A_4 & A_1 \end{bmatrix} \begin{vmatrix} E_{\parallel 0} \\ E_{\perp 0} \end{vmatrix} \quad (1)$$

The Stokes vector $|\mathbf{V}|$, which completely characterizes the intensity and polarization of a light ray, is defined in terms of time averages of the electric field components of an electromagnetic wave:

$$
\begin{aligned}
I &= \left\langle E_{\parallel}E_{\parallel}^{*} + E_{\perp}E_{\perp}^{*} \right\rangle \\
Q &= \left\langle E_{\parallel}E_{\parallel}^{*} - E_{\perp}E_{\perp}^{*} \right\rangle \\
U &= \left\langle E_{\parallel}E_{\perp}^{*} + E_{\perp}E_{\parallel}^{*} \right\rangle \\
V &= \left\langle i(E_{\parallel}E_{\perp}^{*} - E_{\perp}E_{\parallel}^{*}) \right\rangle
\end{aligned}
\quad \text{or} \quad
\left| \begin{matrix} I \\ Q \\ U \\ V \end{matrix} \right| =
\left| \begin{matrix}
\left\langle E_{\parallel}E_{\parallel}^{*} + E_{\perp}E_{\perp}^{*} \right\rangle \\
\left\langle E_{\parallel}E_{\parallel}^{*} - E_{\perp}E_{\perp}^{*} \right\rangle \\
\left\langle E_{\parallel}E_{\perp}^{*} + E_{\perp}E_{\parallel}^{*} \right\rangle \\
\left\langle i(E_{\parallel}E_{\perp}^{*} - E_{\perp}E_{\parallel}^{*}) \right\rangle
\end{matrix} \right| = |\mathbf{V}|. \qquad (2)
$$

Here I is total intensity, Q – polarization at 0° or 90° to the scattering plane, U – polarization at ±45° to the scattering plane, and V – left or right circular polarization. The transformation of an incident four-component Stokes vector $|\mathbf{V}_0|$ by a scattering matrix [S] gives a final Stokes vector $|\mathbf{V}_f|$ where $|\mathbf{V}_f| = [S]\,|\mathbf{V}_0|$ or

$$
\left| \begin{matrix} I_f \\ Q_f \\ U_f \\ V_f \end{matrix} \right| =
\left| \begin{matrix}
S_{11} & S_{12} & S_{13} & S_{14} \\
S_{21} & S_{22} & S_{23} & S_{24} \\
S_{31} & S_{32} & S_{33} & S_{34} \\
S_{41} & S_{42} & S_{43} & S_{44}
\end{matrix} \right| \cdot
\left| \begin{matrix} I_0 \\ Q_0 \\ U_0 \\ V_0 \end{matrix} \right|. \qquad (3)
$$

When the values of the Stokes vector (2) are inserted into the matrix representation (3) one gets the general form of the scattering matrix [4]. Any component of the electric field vector **E** can be written explicitly in terms of its phase E, amplitude a, wave number k, and frequency ω. We have $E(z) = a \exp(-i\varepsilon) \exp[-i(kz - \omega t)]$. This vector is used to get the matrix that gives a direct relationship between the amplitudes a_i ($i = j$ or k) and the phase difference $\delta = \varepsilon_j - \varepsilon_k$. Then $a_k a_k^* = |a_k|^2$; $\langle a_j a_k^* + a_k a_j^* \rangle = |a_j||a_k| \sin\delta$ and $i/2(a_j a_k^* - a_k a_j^*) = |a_j||a_k| \cos\delta$, and we get:

$$
[S] = \begin{bmatrix}
\frac{1}{2}(a_1^2 + a_2^2 + a_3^2 + a_4^2) & \frac{1}{2}(-a_1^2 + a_2^2 - a_3^2 + a_4^2) & (a_3a_2 - a_4a_1)\cos\delta & -(a_2a_3 + a_4a_1)\sin\delta \\
\frac{1}{2}(-a_1^2 + a_2^2 + a_3^2 - a_4^2) & \frac{1}{2}(a_1^2 + a_2^2 - a_3^2 - a_4^2) & (a_3a_2 - a_4a_1)\cos\delta & -(a_2a_3 + a_4a_1)\sin\delta \\
(a_2a_4 + a_3a_1)\cos\delta & (a_2a_4 - a_3a_1)\cos\delta & (a_2a_1 + a_3a_4)\cos\delta & -(a_2a_1 - a_3a_4)\sin\delta \\
(a_2a_4 + a_3a_1)\sin\delta & (a_2a_4 - a_3a_1)\sin\delta & (a_2a_1 + a_3a_4)\sin\delta & (a_2a_1 + a_3a_4)\cos\delta
\end{bmatrix}. \quad (4)
$$

The general scattering matrix given above applies to any system of particles. However, it is convenient to divide the particles into two rather broad ranges depending on the ratio of wavelength λ to particle size d. The two classes of scattering particles are called Rayleigh and Mie particles.

2.1.1. Rayleigh scattering

Particles smaller than the wavelength λ of the incident radiation are called "small particles" or Rayleigh particles. For this condition ($d << \lambda$) the scattering matrix can be exactly calculated. It has the form:

$$
[S] = \text{constant} \begin{bmatrix}
(1+\cos^2\theta) & \sin^2\theta & 0 & 0 \\
\sin^2\theta & (1+\cos^2\theta) & 0 & 0 \\
0 & 0 & \cos\theta & 0 \\
0 & 0 & 0 & \cos\theta
\end{bmatrix}.
$$

The differential scattered light intensity is contained in the S_{11}, matrix element and depends on the size, optical constants, and number density of the scatterers. Small particles which have anisotropic optical constants may have additional non-zero-off-diagonal matrix elements [5].

2.1.2. Mie scattering

Particles larger than the wavelength of the incident radiation are called "large particles" or Mie particles. In this case, only a few particle configurations can be treated exactly by theory. They are spherical particles, the infinite cylinder, and the infinite slab. Some variations of spherical particles, the spherical shell and spherical particle with optical activity, have been recently treated theoretically [1].

In general, signals from uniform sized and shaped (monodispersed) Mie particles are highly structured, the phase of the structure depending on the particle size, shape, and index of refraction. Therefore, small variations from monodisperseness can wash out the structure completely, an effect which is well known for the differential scattered light intensity signal from polydispersed systems.

2.1.3. Biological particles

Biological particles are generally complex arrangements of various shaped structures [6], often having large size distributions resulting in relatively little structure in the differential scattered light intensity S_{ii}. However, complex structure does occur on other S_{ij} signals, as shown by our preliminary work with selected biological systems. The matrix elements we studied in this experiment are:

$$S_{11} = \frac{1}{2}(a_1^2 + a_2^2 + a_3^2 + a_4^2)\,,$$

$$\frac{S_{12}}{S_{11}} = \frac{\frac{1}{2}(-a_1^2 + a_2^2 - a_3^2 + a_4^2)}{\frac{1}{2}(a_1^2 + a_2^2 + a_3^2 + a_4^2)} = \frac{-a_1^2 + a_2^2 - a_3^2 + a_4^2}{a_1^2 + a_2^2 + a_3^2 + a_4^2}\,,$$

$$\frac{S_{13} + S_{33}}{S_{11} + S_{31}} = \frac{(a_3a_2 - a_4a_1 + a_2a_1 + a_3a_4)\cos\delta}{\frac{1}{2}(a_1^2 + a_2^2 + a_3^2 + a_4^2) + (a_2a_4 - a_3a_1)\cos\delta}\,,$$

$$S_{34}^* = \frac{S_{14} + S_{34}}{S_{11} + S_{13}} = \frac{(a_2a_3 + a_3a_4)\cos\delta - (a_2a_3 + a_4a_1)\sin\delta}{\frac{1}{2}(a_1^2 + a_2^2 + a_3^2 + a_4^2) + (a_2a_3 + a_4a_1)\cos\delta}\,.$$

Theoretically the S^*_{34} matrix combination is unique in comparison to other matrix elements. The presence of the phase terms $\cos\delta$ and $\sin\delta$ can cause a sign change in S^*_{34} for very small changes of δ. The larger number of possibilities for zeros and oscillations about zero contribute to the larger structure on the S^*_{34} signal as compared to the other matrix elements and matrix element combinations listed above. The $S^*_{34}(\theta)$ signal can be zero at various θ even if no coefficients are zero. Among other matrix elements the S^*_{34} is highly structured and thus can be an excellent probe of scatterers.

3. CONCLUSIONS

In this introductory research work we have characterized the bio-material with Mueller matrix optical polarimetry. The images and matrix elements with Mueller calculation provides a comprehensive information of samples due to which is possible to combine all the necessary parameters for describing beam of light into a single image. The resultant describing the light beam is simply the four-parameter Stokes vector and determined by measuring a flux transmitted through a set of polarization optics, polarization generating optics provide linear and circular polarized light to sample and polarization analyzing optics collect polarized output light from sample to detecting devices. The characteristic Mueller matrix in all experiments contains 16 elements, having total 49 intensity measurements at different polarization states. In practice, all 16 elements are independent and other depending on the symmetry and certain proprieties of the optical medium.

The Stokes-Mueller polarization images of transmitted intensity along with degree of polarization provide fingerprint of the turbid medium. Careful analysis of images and degree of polarization, differentiation of scatterer in term of its concentration, size, shape and orientation is possible. The linear polarization preservation is dominant over circular due to scatterer density rather than size of the particle in the medium. We have seen that linearly and circularly polarized light propagates differently for turbid samples.

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