ROBUST STABILITY OF AIR-TUNNEL MODEL

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ABSTRACT

Text deals briefly with robust stability which is one of the characteristics of a robust system. This attribute is described in the text; there are investigations carried out of robust stability by complex characteristics on taken models (continuous and discrete) and also on continuous and discrete feedback loop. These results are compared with the results of the physical model. All simulations, computations and results are obtained with MATLAB-Simulink and B&R Automation Studio.

1. INTRODUCTION

In this article we will deal more closely with the observation of robust stability of the airtunnel model. We will obtain its mathematical description, define the uncertainties of its parameters, and find boundary values for the complex characteristic of the model in question. We will further inspect the differences of this attribute for continuous and discrete control loop. The results obtained will be later verified on the physical model.

2. PROBLEM

Our task is to control the model of the air-tunnel which is situated in our laboratory and it contains non-linearity. This non-linearity is caused by barriers which are in the tunnel (creating turbulences) and we use ventilator (non-linear element) to drive the air into the tunnel and to measure the air-flow. We control this tunnel in basic feedback loop around operating point 30 % because in this part of the operating range the influence of the non-linearity is minimal. We will use algorithm PID (3) or PSD (4) for the controller with the use of the Ziegler-Nichols tuning method.

3. ROBUST STABILITY

Dictionary defines the expression of robustness as a force, potency or "attribute to be strong". We speak about the dynamics of closed loop which enables the controller to execute an activity according to the requirements in a chosen system; also in a case when a system which changes its properties is concerned.

One of the primary conditions of robustness of the closed loop is the condition of robust stability. This term is bounded with uncertainty or indefinity of system and says that: The

closed loop must be stable for all possible changes of parameters of the plant which are defined by the volume of this uncertainty of the system. For example we have plant

$$G(s) = \frac{k}{as+1} \tag{1}$$

Whose transfer function contains parameter which holds: $a \in \langle a_{\min}; a_{\max} \rangle$. The definition says that the closed loop must be stable for all possible values of parameter *a* from the taken interval. By 'stable' we understand that the complex characteristic of open-loop control circuit $F_0(j\omega)$ with growing frequency ($\omega \ge 0$) does not intersect the point (-1; j0) and passes to the right of this point.



Figure 1: Robust stability

3.1. AIR-TUNNEL MODEL

To verify the robust stability we have chosen a model of air-tunnel. This contains the proper air-tunnel which has at the entrance a ventilator attached which has the function of an actuator (controlled by PLC B&R in a range $0\div10V$ with 12-bit D/A converter). This ventilator drives air into the air-tunnel. The driving ventilator exhibits a $5\div8$ % dead zone at the initial point of the coordinates. At the end of the tunnel there is a second ventilator which has the function of air-flow sensor and this linear motion is converted into rotational. By means of optoelectronic sensor the rotational motion is converted to electric signal it is further processed by additional electronics which sends this processed signal to 12-bit A/D converter in PLC B&R. The PLC B&R is connected to PC by Ethernet where runs the program MATLAB-Simulink by which the whole task is carried out. Basic scheme of the given model is presented in Fig. 2 (left). On the basis of experience with the model we will assign the order of the mathematical model to 3^{rd} and its transfer function is

$$G(s) = \frac{n_1 s + n_0}{s^3 + a_2 s^2 + a_1 s + a_0}$$
(2)

We assign the parameters of the model with estimation and adjust them with an experimental method by comparing the step response of the physical model without using the identification algorithm. Where $n_1 = 0.20$; $n_0 = 2,80$; $a_1 = 10,00$; $a_2 = 10,00$; $a_3 = 2,00$ and for each of these parameters we accept an uncertainty of ± 10 % from the real system.

In Fig. 2 (right) we can see the complex characteristic of the model with nominal values $G(j\omega)$ and then the characteristics $G(j\omega)_{S\min}$, $G(j\omega)_{S\max}$, $G(j\omega)_{Z\min}$ and $G(j\omega)_{Z\max}$ (zero order hold, sampling time 0.2 s). These characteristics were obtained by graphical comparison of all the possible combinations of the maximal and minimal values of uncertain parameters.



Figure 2: The model of the real system of the air-tunnel (left) and complex characteristic of the model (right).

3.2. CONTROLLER AND CLOSED-LOOP CONTROL

By using the controller to control the air-tunnel model the situation of complex characteristic changes markedly because the controller changes frequency properties of open-loop control circuit. For comparison we will use the continuous (3) and discrete (4) PID controller with filtration of derivation:

$$F_R(s) = K \left[1 + \frac{1}{T_i s} + \frac{T_d s}{\frac{T_d}{N} s + 1} \right]$$
(3)

$$F_{R}(z) = K \left[1 + \frac{z^{-1}T}{T_{i}(1-z^{-1})} + \frac{T_{d}}{T} \left(1 - e^{-\frac{NT}{T_{d}}} \right) \frac{1-z^{-1}}{1-e^{-\frac{NT}{T_{d}}} z^{-1}} \right]$$
(4)

And the parameters gained by the modified Ziegler-Nichols method are K = 8.00; $T_i = 5.00$; $T_d = 0.25$; N = 3.00; T = 0.20 s (sampling time).



Figure 3: Kharitonov's plot of characteristic polynomial of the system. On the right side there is a detail around the beginning of the system of coordinates.

On the basis of Fig. 3 we can claim that uncertainties of the system will not cause the instability of the continuous control loop. This claim does not have to be true for the discrete control loop. Because of that we have to verify the behaviour of the discrete control loop.

In Fig. 4 (left) we can see that the controller caused a change in the shape of the original complex characteristic $G(j\omega)$. The shape of $F_0(j\omega)_{Z_{\text{max}}}$ indicates instability of the control loop. For this reason it is necessary to design a controller which stabilizes the control loop. The parameters of the controller are as follows: K = 1.00; $T_i = 3.00$; $T_d = 0.25$; N = 3.00; T = 0.20 s (sampling time). From the shape of the complex characteristic of $F_0(j\omega)_{Z_{\text{stab}}}$ (in Fig. 4 left) it is obvious that this change of parameters stabilized the given control loop.



Figure 4: Nyquist plot (left) of the open-loop system (controller without anti-windup or the restriction of the actuator). Step response (right) of the closed-loop system (controller without anti-windup or the restriction of the actuator).



Figure 5: The comparison of control process in a real system and control loop with discrete controller (4) and continuous model (contains anti-windup and restriction of the actuator and the 12-bit A/D and D/A converter).

4. CONCLUSION

Our task was to design a controller for a real system whose mathematical description we know only approximately. In the beginning we found the worst alternative of complex characteristic for given mathematical model. The worst alternative could cause that the controller designed by us will fail to assure the stability of the control loop or it could even cause the instability of the loop. When we know that the $F_0(j\omega)_{Z \max}$ is unstable, we were forced to adjust the parameters of the controller so that the characteristic of the open loop fulfils the condition of robust stability. We can observe this change in Fig. 4 (left) and the consequences on the step response in Fig. 4 (right). Fig. 5 is the most important result because we can see there the application of the discrete controller (4) designed by us on the real model of the air-tunnel and its comparison with control loop which contains a discrete controller (4) and a continuous system with nominal values of uncertain parameters. From the results obtained in Fig. 4 (left) it is not clear whether the instability was caused by utilization of PSD controller instead of PID one or by neglecting the dead time which is caused by discretization of the continuous model.

REFERENCES

- Pivoňka, P., Schmidt, M.: Comparative Analysis of Discrete Derivative Implementations in PID Controllers. In Systems Theory and Applications. vol.2. Řecko: WSEAS, 2007. s. 33-37. ISBN: 978-960-8457-90-4.
- [2] Dobias, M.: Robustnost regulátorů [diplomová práca], VUT FEKT Brno, 2009.
- [3] Mackenhort, U.: Robust Control Systems Theory and Case Studies, Springer-Verlag Berlin Hiedelberg, 2004