

# COMPARISON OF PMSM AND INDUCTION MACHINE BASED ON GENERALIZED SIZING EQUATION

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## ABSTRACT

This article is aimed to compare of induction machine and synchronous machine with permanent magnet. This comparison is based on generalized sizing equation. Comparison will be sized of machine – diameter of air gap and length. For comparison, we will suppose, that both type of machine have the same efficiency and output power. In this article there is calculated only with sizing equation and there isn't calculated with another aspect of design of electrical machine

## 1. GENERALIZED SIZING EQUATION

Generalized sizing equation was first published in [1] for induction machine. Sizing equation was modified to form, which can be used on various type of machine [2].

Generalized sizing equation for radial machine is [2]

$$P_r = \frac{1}{1 + K_\phi} \cdot \frac{m}{m_1} \cdot \frac{\pi}{2} \cdot K_e \cdot K_i \cdot K_p \cdot \eta \cdot B_g \cdot A \cdot \frac{f}{p} \cdot \lambda_0^2 \cdot D_0^2 \cdot L_e \quad (1)$$

where  $K_\phi$  is ratio of electrical loading on rotor and stator,  $m$  is number of phases of machine,  $m_1$  is number of phases of each stator if machine has more than one stator,  $K_e$  is EMF factor,  $K_i$  is current waveform factor,  $K_p$  is electrical power waveform factor,  $\eta$  is efficiency,  $B_g$  is peak value of flux density in air gap,  $A$  is total electrical loading (both stator and rotor),  $f$  is frequency,  $p$  is number of pole pairs,  $\lambda_0$  is ratio of the air-gape diameter and outer surface diameter,  $D_0$  is diameter of outer surface of machine and  $L_e$  is effective stack length of the machine .

Parameter  $K_\phi$  depends on ratio of electric loading of stator and rotor. In the case that machine haven't rotor winding,  $K_\phi$  is zero.

Parameter  $K_e$  incorporates with winding factor  $k_w$  and per unit portion of total air gap area and area spanned by salient pole. One of way how to calculate this parameter is to compare general equation of peak value of EMF voltage (2) and equation of EMF voltage  $E_f$  for a machine.

$$E_f = K_e \cdot N_1 \cdot B_g \cdot \frac{f}{p} \cdot \lambda_0 \cdot D_0 \cdot L_e \quad (2)$$

where  $N_1$  is number of turn in one phase. Equation (2) is valid for machine with radial air gap.

Parameter  $K_i$  can be calculated from (3)

$$K_i = \frac{I_{pk}}{I_{rms}} = \sqrt{\frac{1}{T} \cdot \int_0^T \left( \frac{i(t)}{I_{pk}} \right)^2 dt} \quad (3)$$

where  $T$  is period of current waveform,  $I_{pk}$  is peak value of current,  $I_{rms}$  is RMS value of current and  $i(t)$  is expression of current waveform. If current waveform is sinusoidal, from (3) can be calculated that  $K_i = \sqrt{2}$ .

Parameter  $K_p$  can be calculated from (4)

$$K_p = \frac{1}{T} \int_0^T f_e(t) \cdot f_i(t) dt \quad (4)$$

where  $f_e(t)$  is normalized EMF waveform,  $f_i(t)$  is normalized current waveform. In case that both waveforms (EMF and current) have sinusoidal shape, from (4) can be calculated that  $K_p = \frac{1}{2} \cdot \cos \varphi$

### 1.1. GENERALIZED SIZING EQUATION FOR IM

For  $K_e$  calculation we can start from (2) and equation for RMS value of EMF voltage for induction machine (6)

$$E_{fRMS} = \sqrt{2} \cdot \pi \cdot N_1 \cdot f_1 \cdot \phi \quad (5)$$

$$E_{fRMS} = \sqrt{2} \cdot \pi \cdot N_1 \cdot \frac{f_1}{p} \cdot B_g \cdot L_e \cdot D_g \cdot k_w \quad (6)$$

When we compare equation (2) and (6), we can express that coefficient  $K_e = 2 \cdot \pi \cdot k_w$ .

For induction machine is number of phases of machine and number of phases of each stator identically  $\frac{m}{m_1} = 1$ . When we introduce coefficients  $K_e$ ,  $K_i$ ,  $K_p$  to (1), introduce

$\lambda_0 = \frac{D_g}{D_0}$  and if the new equation is simplified, we will get the general sizing equation for induction machine (7)

$$P_r = \frac{1}{1 + K_\phi} \cdot \frac{\pi^2}{2} \cdot k_w \cdot \sqrt{2} \cdot \cos \varphi \cdot \eta \cdot B_g \cdot A \cdot \frac{f}{p} \cdot D_g^2 \cdot L_e \quad (7)$$

## 1.2. GENERALIZED SIZING EQUATION FOR PMSM

For RMS value of EMF voltage of PMSM calculation we can use equation (8)

$$E_{fRMS} = \sqrt{2} \cdot \pi \cdot N_1 \cdot f_1 \cdot \phi \quad (8)$$

$$E_{fRMS} = \sqrt{2} \cdot \pi \cdot N_1 \cdot \frac{f_1}{p} \cdot B_g \cdot L_e \cdot D_g \cdot k_w \cdot \alpha_i \quad (9)$$

where  $\alpha_i$  is effective relative magnet width. On the base of comparison of (2) and (9) we can calculate that  $K_e = 2 \cdot \pi \cdot k_w \cdot \alpha_i$ .

For PMSM we calculated, that power factor is 1 and because there isn't rotor winding, coefficient  $K_\phi = 0$ . After introduce to (1), we will get the generalized sizing equation for PMSM

$$P_r = \frac{\pi^2}{2} \cdot k_w \cdot \alpha_i \cdot \sqrt{2} \cdot \eta \cdot B_g \cdot A \cdot \frac{f}{p} \cdot D_g^2 \cdot L_e \quad (10)$$

## 2. COMPRAISON OF IM AND PMSM

Now we can compare sizing equation for IM (7) and PMSM (10)

$$\frac{1}{1+K_\phi} \frac{\pi^2}{2} k_w \sqrt{2} \cos \varphi \eta B_g A \frac{f}{p} D_g^2 L_e = \frac{\pi^2}{2} k_w \alpha_i \sqrt{2} \eta B_g A \frac{f}{p} D_g^2 L_e \quad (11)$$

For final comparison of both type of machine, we must put in some value to (11). First value is air gap flux density and linear current density. The permitted value of this parameter is in Tab.1. For simplification we can suppose, that value of flux density and linear current density is identical.

	PMSM	IM
$B_g$ [T]	0,8-1,05	0,7-0,9
$A$ [A.m <sup>-1</sup> ]	35000-65000	30000-65000

**Tab. 1.** Permitted value of air gap flux density  $B_g$  and linear current density  $A$  [3]

For another simplification we can suppose, that IM and PMSM have the same value of  $k_w$ ,  $f$ ,  $p$ ,  $P_r$  and  $\eta$ .

If we simplify (11), we can get the comparison of sizing equation (12)

$$\underbrace{\frac{1}{1+K_\phi} \cdot \cos \varphi \cdot D_g^2 \cdot L_e}_{IM} = \underbrace{D_g^2 \cdot L_e \cdot \alpha_i}_{PMSM} \quad (12)$$

For better comparison we must set up value  $K_\phi$ . For introduction of this value we can use Tab. 2.

cos $\varphi$	0,65	0,7	0,75	0,8	0,85	0,9	0,95
$K_\phi$	0,74	0,77	0,82	0,86	0,9	0,95	0,985

**Tab. 2.** Value of  $K_\phi$  for different value of power factor [2]

Now, we can come to ratio of  $D_g^2 \cdot L_e$  for PMSM and IM

$$\frac{(D_g^2 \cdot L_e)_{PMSM}}{(D_g^2 \cdot L_e)_{IM}} = \frac{\cos \varphi}{(1 + K_\phi) \cdot \alpha_i} \quad (13)$$

From equation (13) we can see, that PMSM can be smaller for the same power then induction machine.

### 3. CONCLUSION

On the base of this comparison, we can say, that we can construct PMSM smaller then IM in the same value of output power. In this compression wasn't calculated with another aspect of design of this type machine. In reality, ratio of  $D_g^2 \cdot L_e$  for PMSM and IM will be higher then value calculated base on equation (13).

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### REFERENCES

- [1] V. B. Honsinger, "Sizing equations for electrical machinery," IEEE Trans. Energy Conversion, vol. EC-2, pp. 116–121, Mar. 1987.
- [2] S. Huang, J. Luo, F. Leonardi, T. A. Lipo, "A General Approach to Sizing and Power Density Equations for Comparison of Electrical Machines," IEEE Transactions on Industry Applications, IEEE Trans. IA-34, No.1, pp.92-97, 1998
- [3] Pyrhönen J., Jokinen T., Hrabovcová V.: Design of Rotating Electrical Machine, John Wiley and Sons, Chichester, 2008