NUMERICAL APPROACH TO TRANSIENT ANALYSIS OF PIEZOCERAMIC SENSOR

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ABSTRACT

This paper presents the numerical approach and results for a piezoceramic sensor for acoustic emission sensing. A method for the analysis of piezoelectric media based on finite element calculations is presented. The method is applied to the transient analysis of piezoelectric sensor.

1 INTRODUCTION

Robust numerical simulations of advanced structures will result in the development of designs that perform better than their passive counterparts while costing less. In order to optimize the design of complicated structures that are subjected to critical loading, dynamic simulation is necessary. The finite-element method is very attractive since it can be applied to any geometry for any set of material properties and loading conditions as long as the appropriate constitutive relationships and equilibrium conditions are met. Since the method is not restricted by size, one can use the so called zoom feature in finite-element meshing to use different-size elements to describe device. Many researchers have used the finite-element method for modeling piezoelectric sensors and actuators since the 1970s. The first finite element formulation was proposed by Allik and Hughes [1]. A comprehensive paper was written by Lerch [2] on the simulation of piezoelectric devices that included time domain modeling. The numerical approach presented may be used in CAE models for nondestructive testing sensors.

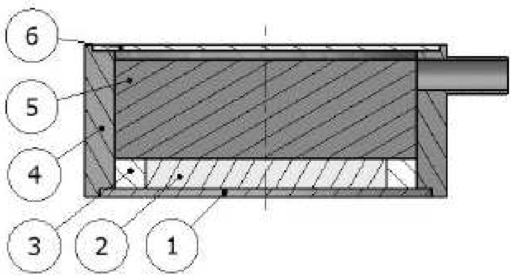


Fig. 1: *Piezoceramic sensor in detail (1. membrane, 2. piezoceramic segment, 3. insulated ring, 4. case base, 5. damping material, 6. case cover)*

2 FORMULATION OF PIEZOELECTRIC ELEMENTS

Piezoelectric materials are anisotropic and the elastic field in such materials is coupled with the electric field. Finite element equations for piezoelectric materials have already been formulated in many papers. A finite-element formulation is presented for modeling the dynamic response of piezoelectric ceramic sensors.

The base part of sensor is sensing segment. In our case the object of study is a piezoceramic disc, see fig. 2. The finite element equations (motion equations) used in the calculation of dynamic response are given by [3]

$$\begin{vmatrix} M & 0 \\ 0 & 0 \end{vmatrix} \begin{vmatrix} \ddot{U} \\ \ddot{\Phi} \end{vmatrix} + \begin{vmatrix} D & 0 \\ 0 & 0 \end{vmatrix} \begin{vmatrix} \dot{U} \\ \dot{\Phi} \end{vmatrix} + \begin{vmatrix} K_{uu} & K_{u\phi} \\ K_{u\phi}^T & K_{\phi\phi} \end{vmatrix} \begin{vmatrix} U \\ \Phi \end{vmatrix} = \begin{vmatrix} F \\ Q \end{vmatrix}, \tag{1}$$

where $\{U\}$ is a vector of nodal displacements, $[K_{uu}]$ is mechanical stiffness matrix, $[K_{u\Phi}]$ is piezoelectric coupling matrix, $[K_{\Phi\Phi}]$ is dielectric stiffness matrix, $[M]$ is mass matrix, $\{F\}$ is mechanical forces and $\{Q\}$ is electrical charges. Damping matrix is defined $[D] = \alpha[M] + \beta[K_{uu}]$, where α and β are called Rayleigh coefficients. Damping constants α and β are determined empirically by examining critical damping at two different frequencies. The finite element (FE) equations are transformed to H-form [3, 4], where the potential in the nodes of the elements are condensed out of the FE equations, and instead the voltage V is introduced

$$\begin{vmatrix} M & 0 \\ 0 & 0 \end{vmatrix} \begin{vmatrix} \ddot{U} \\ \ddot{V} \end{vmatrix} + \begin{vmatrix} D & 0 \\ 0 & 0 \end{vmatrix} \begin{vmatrix} \dot{U} \\ \dot{V} \end{vmatrix} + \begin{vmatrix} H_{uu} & H_{u\phi} \\ H_{u\phi}^T & H_{\phi\phi} \end{vmatrix} \begin{vmatrix} U \\ V \end{vmatrix} = \begin{vmatrix} F \\ Q \end{vmatrix}. \tag{2}$$

The basic concept of matrix condensation is based on Gaussian elimination solution of equations for unknowns. After assembling all element matrices, the system dynamic equation is written as

$$[M]\{\ddot{U}\} + [D]\{\dot{U}\} + [H^*]\{U\} = \{F^*\}, \tag{3}$$

where

$$[H^*] = [H_{uu}] - [H_{u\phi}][H_{\phi\phi}]^{-1}[H_{u\phi}^T],$$

$$[F^*] = [F] - [H_{u\phi}][H_{\phi\phi}]^{-1}\{Q\}$$

and electrical voltage can be recovered by the relationship

$$V = [H_{\phi\phi}]^{-1}\left(\{Q\} - [H_{u\phi}^T]\{U\}\right).$$

3 TIME-HISTORY ANALYSIS: A DIRECT INTEGRATION

In direct integration the equations of motion (2) are directly integrated using a step-by-step numerical procedure without transformation of the equations to a different form. The step-by-step integration procedures provide an approximate solution at n discrete time intervals $0, \Delta t, 2\Delta t \dots t, t+\Delta t, T$, where T is duration of the input motion or loading and $\Delta t = T/n$. Many numerical integration procedures have been developed. Direct integration methods are generally classified as either explicit or implicit. The basic concept common to most explicit methods is to write the equations of motion for the beginning of the time step, approximate the initial velocity and acceleration terms by finite-difference expressions, and then solve for response at the end of time step. This way the response values calculated in each step depend only on quantities obtained in the preceding step. Therefore, the numerical process proceeds directly from one step to next. Explicit methods are very convenient, but they are only conditionally stable and will "blow up" if time step is not sufficiently small. In an implicit method the expressions for new values at $t+\Delta t$ use equilibrium equations at $t+\Delta t$, and thus include one or more values pertaining to that same step.

3.1 CENTRAL DIFFERNECE METHOD

In our calculation scheme central difference (explicit) method was used. The reason why is equal time step, because the data of simulations are then used for FFT computation. This method is a very simple explicit method that uses the following finite-difference expressions for approximation of the initial velocity and acceleration terms

$$\begin{aligned} \{\ddot{u}_t\} &= [\{u_{t-\Delta t}\} - 2\{u_t\} + \{u_{t+\Delta t}\}] / \Delta t^2 \\ \{\dot{u}_t\} &= [-\{u_{t-\Delta t}\} + \{u_{t+\Delta t}\}] / (2\Delta t) \end{aligned} \quad (4)$$

The displacement solution for time step $t+\Delta t$ (i.e. $\{u_{t+\Delta t}\}$) is obtained by considering the equations of motion (3) at time step t

$$[M]\{\ddot{U}_t\} + [D]\{\dot{U}_t\} + [H^*]\{U_t\} = \{F_t^*\} \quad (5)$$

Substituting (4) into (5), leads to

$$\left(\frac{1}{\Delta t^2}[M] + \frac{1}{2\Delta t}[D]\right)\{U_{t+\Delta t}\} = \{F_t^*\} - \left([H^*] - \frac{2}{\Delta t^2}[M]\right)\{U_t\} - \left(\frac{1}{\Delta t^2}[M] - \frac{1}{2\Delta t}[D]\right)\{U_{t-\Delta t}\}. \quad (6)$$

Concerning stability, the central difference method is conditionally stable: the time step value must be smaller than a critical one. For linear problems, this critical time step Δt_c is

$$\Delta t_c \le 2/\omega_{\max} , \quad (7)$$

where ω_{max} is the highest natural frequency, bounded by the maximum frequency of the individual finite elements.

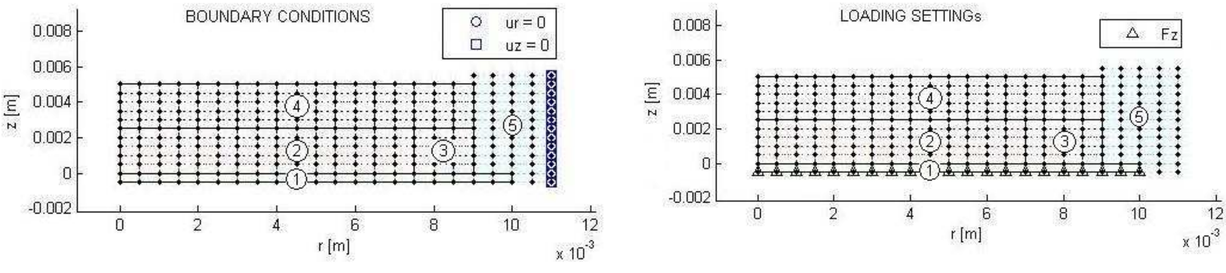
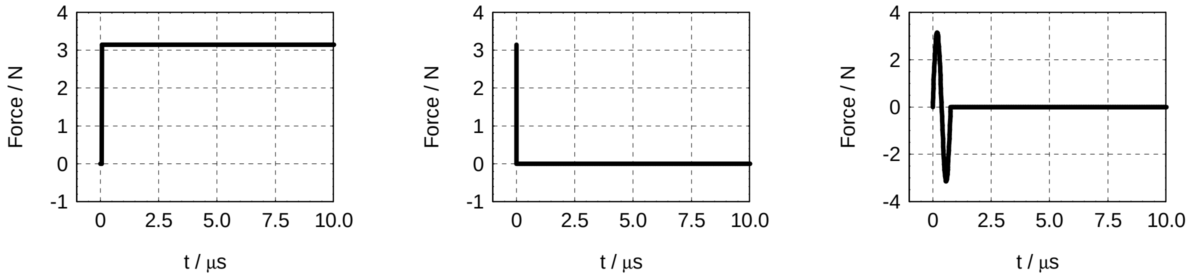


Fig. 2: *Boundary condition and loading of model*

4 NUMERICAL SOLUTIONS AND RESULTS

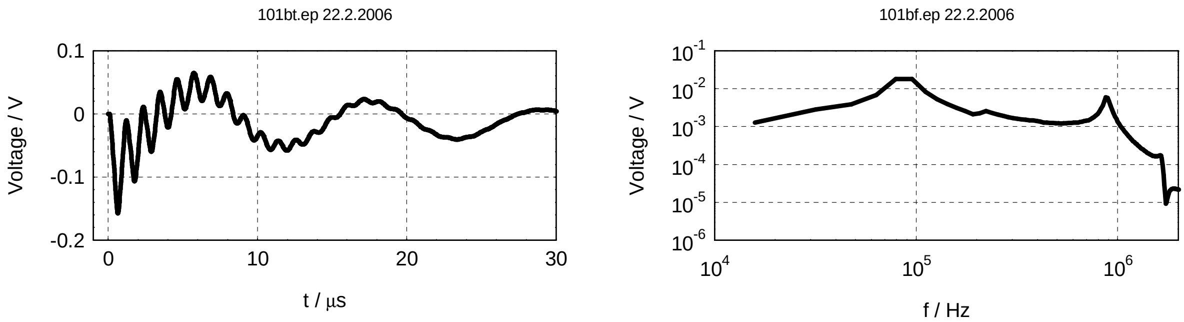
Algorithm was developed in MATLAB. For simplicity, the problem is defined as two dimensional case, axisymmetry task. The object of study was piezoceramic disc (material PZT-27; Ferroperm-piezo) in sensor. A schematic description of the sensor is presented in figure 1. For calculation scheme the linear quadratic elements were used. The accuracy of the transient solution including a piezoelectric sensor also increases with decreasing time step. The smaller the time step, however, the larger the number of iterations required for solution. Therefore, the time step chosen should be small enough for an accurate solution but large enough to minimize iteration. At least ten time steps per period must be taken for accuracy. The time step used is 7×10^{-9} sec.



a) unit step *b) pulse* *c) burst signal*

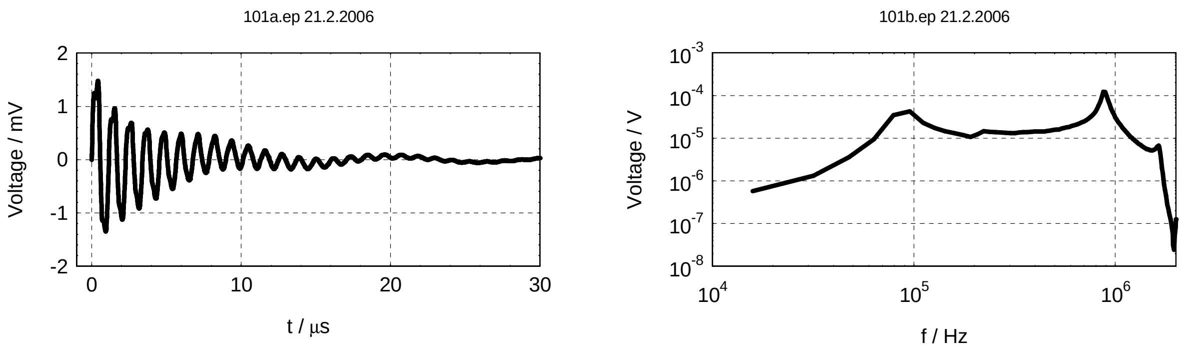
Fig. 3: *Types of load history function used in the numerical models.*

The boundary condition and loading is shown in fig. 2. Three types of loading (see fig. 3.) were studied: step function, simply pulse and burst signal.



a) time domain *b) frequency domain*

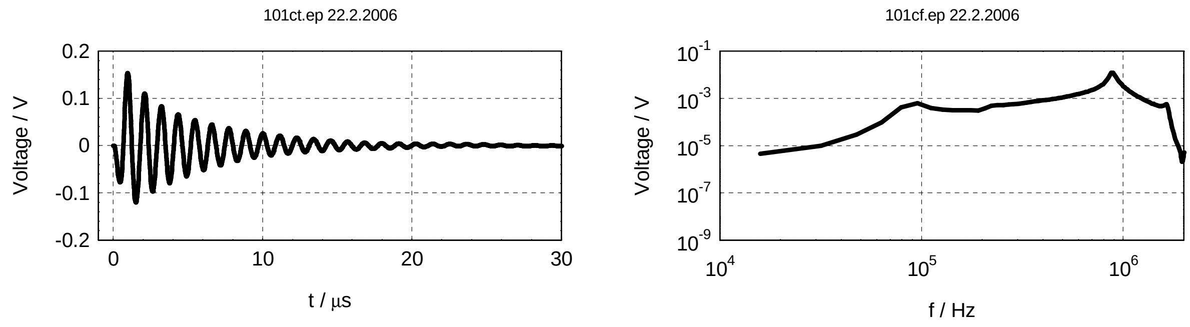
Fig. 4: *Piezoceramic sensor response for unit step excitation.*



a) time domain *b) frequency domain*

Fig. 5: *Piezoceramic sensor response for simply pulse excitation.*

In figures 4–6, the sensor voltage responses are plotted for unit step and simply pulse excitation and a burst pulse, respectively. In all three cases, an initial transient response phase can be noticed, and then damped vibration at the natural frequency of the system and approach to the static steady state value.



a) time domain *b) frequency domain*

Fig. 6: *Piezoceramic sensor response for simply pulse excitation.*

5 CONCLUSION

This paper is concerned with the finite-element modeling of the dynamic response of piezoceramic sensor. Reduction scheme (3) was employed to condense the elastodynamic and electric degrees of freedom. The time-history response was calculated by a direct integration algorithm (central difference method) to accommodate piezoelectric constitutive equations [1]. To numerically simulate the dynamic response of sensor the example of a prototype of acoustic emission sensor was used.

The transient analysis is important to knowledge of frequency characteristic of sensor. So the In order to verify the proposed analysis, finite-element results was compared with known analytical results for various dynamic loads. The agreement is good. The influence of dynamic loading on the transient response of sensor is demonstrated. The results indicate that a system would generally oscillate during excitation with varying amplitudes, and at a frequency that corresponds to the natural frequency of the system, but voltage oscillations would gradually reach a steady state value because of structural damping.

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