# QUADRATURE MIRROR FILTER BANK AND SIGMA-DELTA MODULATION

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**ABSTRACT**

A quadrature mirror filter banks are the most frequently realized by FIR filters. The input signal is decomposed into some frequency subbands by these filters. In the case of dual channel filter banks this subbands are two. A subbands coding can be done in individual subbands. By this process the subbands signals are distorted. This distortion gets into reconstructed output signal. In the case of replacement FIR filters in decomposition section by sigma-delta modulators a new structure may be designed, which is more resistant to this type of distortion.

## 1 INTRODUCTION

Generally the decomposition filter bank is used to decomposition of signal into individual subbands and the reconstruction filter bank is used to reconstruction of output signal from individual subbands. The basic type is dual channel filter bank whose outputs are two subbands with same width. Dual channel filter bank is composed of low pass filter and high pass filter. Next type is multi channel filter bank which consist more filters in comparison with dual channel filter bank. The whole spectrum of input signal is decomposed into some subbands with same width. Multi channel filter bank is composed of one low pass filter, some band pass filter a one high pass filter.

In the case of individual subbands signals are quantized, they are distorted. So that the quantization can be at least number of bits it is necessary so the transfer of useful signal stays without change and so inserted quantization noise is more suppressed in reconstruction filter bank. Because reconstruction filter bank is composed of one high pass filter and one low pass filter, it is suitable so whole added quantization noise is into the part of spectrum which is more suppressed by passage of reconstruction filter bank. The used quantizer adds uniform spread quantization noise into spectrum of subbands signal. It is a possibility to merging decomposition filter bank and quantizer into one function block and this block is realized by sigma-delta modulator which has the same impulse response as replaced low pass filter and high pass filter. In sigma-delta modulator the noise is shaped, but its transfer function can be chosen in accordance required property of filter bank. The more suppression of quantization noise by passage throw filter bank allows least number of quantizations layers in preserve same quantizations noises level on filter banks outputs. In this way it gets greater compression ratio.

## 2 QUADRATURE MIRROR FILTER BANK

The most simple realization of dual channel filter bank is twos filters, [2], [3]. First filter is low pass filter and second filter is high pass filter. The block scheme of dual channel filter bank is in fig. 1. The left part of block scheme which contain filters $H_0(z)$ and $H_1(z)$ is decomposition part. The spectrum of input signal $x_n$ is decomposed to high part and low part in the decomposition part. Because the bandwidth is decreased by filtering, the filtered signals can be decimated. The output signals from decomposition filter bank are $x_{0,n}$ and $x_{1,n}$. The right part of block scheme is reconstruction part which reconstruct original signal from subbands signals $x_{0,n}$ and $x_{1,n}$. At first the decimated signals are interpolated to their original sampling frequency and then these signals are filtered by filters $G_0(z)$ and $G_1(z)$. The outputs of both of filters are added and their sum is reconstructed signal $x_n$. If the reconstructed signal is identical with input signal to decomposition filter bank then the filter bank is filter bank with perfect reconstruction. The reconstructed signal can be only delayed.

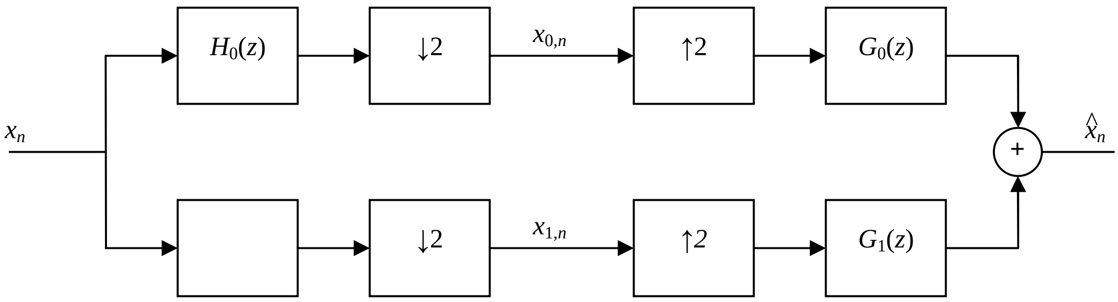


**Fig. 1:** *Block scheme of dual channel filter bank*

The mathematical relations of passage throw structure in fig. 1 are equations (1) and (2).

$$H_0(z)\cdot G_0(z)+H_1(z)\cdot G_1(z)=2z^{-k} \tag{1}$$

$$H_0(-z)\cdot G_0(z)+H_1(-z)\cdot G_1(z)=0 \tag{2}$$

First equation (1) describes the passage of signal throw individual branches without distortion. The time delay can happen because realized filters must be causal. Second equation (2) describes condition for aliasing suppression. In correct design of filter bank the noise signal from decimators in individual branches is each other compensated. This state describe zero on the right hand of equation (2). This equation is fulfilled if next equations (3) and (4) are fulfilled.

$$G_0(z)=H_1(-z) \tag{3}$$

$$G_1(z)=-H_0(-z) \tag{4}$$

## 3 SIGMA-DELTA MODULATOR

The basis of sigma-delta modulator is the delta modulator. In the delta modulation the change of signal is transferred by one bit and therefore the condition of tracing having must be fulfilled for the input signal. Adjusting the delta modulator yields the sigma-delta modulator, [1]. Its block scheme is in fig. 2, where $X(z)$ is the input signal and $Y(z)$ is output signal.

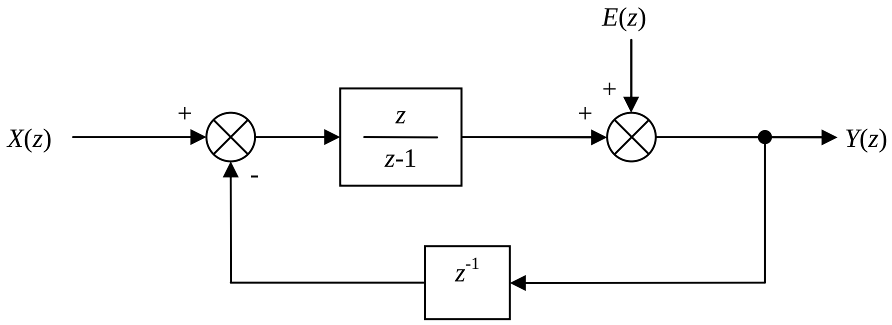


**Fig. 2:** *Linear model of sigma-delta modulator*

The sigma-delta modulator consists of an adder, digital integrator, quantizer and delay element. Input signal $X(z)$ enters the adder which compute the difference between the input and the previous output sample. The difference is gradually accumulated in the digital integrator and then quantized. The quantizer is only one nonlinear component in the sigma-delta modulator, the other components are linear. The analysis of nonlinear circuits is difficult. To simplify the analysis, the quantizer is replaced by the adder, in which quantization noise adder. The resultant linear model block scheme is in fig. 2, where $E(z)$ is the quantization noise. Its value is determined statistically from the number of the quantizer bits. Now the method for linear circuits analysis can be used and compute the transfer function of the input signal and the transfer of quantization noise.

The output signal of this block scheme is expressed by the relation

$$Y(z) = X(z)\cdot\frac{z}{z-1} + Y(z)\cdot z^{-1}\cdot\frac{z}{z-1} + E(z). \tag{5}$$

A simple mathematical arrangement leads to the next equation

$$Y(z) = X(z) + E(z)\left(1 - z^{-1}\right), \tag{6}$$

which express the transfer function of input signal $X(z)$ by the sigma-delta modulator and the transfer function of quantization noise $E(z)$ by the sigma-delta modulator. Input signal $X(z)$ is without change, but the quantization noise is shaped such that it almost does not appear at low frequencies but appears at high frequencies.

## 4 SIGMA-DELTA MODULATOR IN QUADRATURE MIRROR FILTER BANK

If a filter bank with subsequent quantization is used in coding signals then this structure may be replaced by a sigma-delta modulator. Since the dual channel filter bank consists of two filters it is necessary to use two sigma-delta modulators too. For the filters to be used, they must be designed as that the transfer function of the input signal is not constant but has the required frequency characteristic. The digital integrator is replaced by a block with general transfer function *K*(*z*) and a block with transfer function *J*(*z*) is adder to the feedback. The linear model block scheme of sigma-delta modulator is in fig. 3.

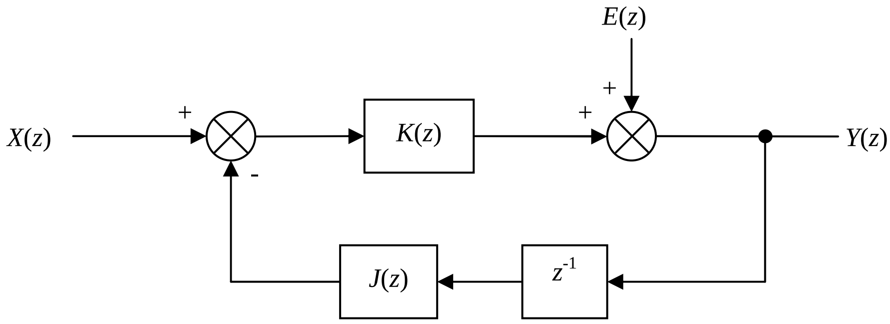


**Fig. 3:** *Linear model of sigma-delta modulator with general transfer function*

The output signal of this block scheme is expressed by the relation

$$Y(z) = X(z) \cdot \frac{K(z)}{1 + z^{-1} \cdot J(z) \cdot K(z)} + E(z) \cdot \frac{1}{1 + z^{-1} \cdot J(z) \cdot K(z)}. \tag{7}$$

The first element expresses the transfer of useful signal and the second element expresses the transfer of quantization noise. For the frequency characteristic to correspond to the characteristic of the filter in the filter bank, their transfer must be same. The condition for transfer function *J*(*z*) is equation (8).

$$J(z) = \frac{K(z) - H(z)}{z^{-1} \cdot K(z) \cdot H(z)}. \tag{8}$$

Substituting transfer function *J*(*z*) into the expression for noise transfer, we get the expression:

$$E(z) \cdot \frac{1}{1 + z^{-1} \cdot J(z) \cdot K(z)} = E(z) \frac{H(z)}{K(z)}. \tag{9}$$

The resulting transfer of system in fig. 3 is

$$Y(z) = X(z) \cdot H(z) + E(z) \frac{H(z)}{K(z)}. \tag{10}$$

From this relation is followed that the transfer of input signal is exactly *H*(*z*) and sigma-delta modulator has the required transfer. The noise transfer depends on the transfer of the input signal too, but it is additionally divided by transfer function *K*(*z*), which can be chosen. Choosing it suitably may lead to the noise being transferred mostly in the stop band of the reconstruction filter bank, and most suppressed in the pass band. Example of transfer function *K*(*z*) is equation (11).

$$K(z) = 0{,}5 + z^{-1} + 0{,}5z^{-2} \tag{11}$$

A complete block scheme of decomposition and reconstruction filter bank is in fig. 4. Both filters in the decomposition part are realized via sigma-delta modulator and the reconstruction part is realized only via filters. The resulting transfer of this whole system is:

$$Y(z) = X(z)\cdot H_0(z)\cdot G_0(z) + X(z)\cdot H_1(z)\cdot G_1(z) + E_0(z)\frac{H_0(z)}{K_0(z)} + E_1(z)\frac{H_1(z)}{K_1(z)}. \qquad (12)$$

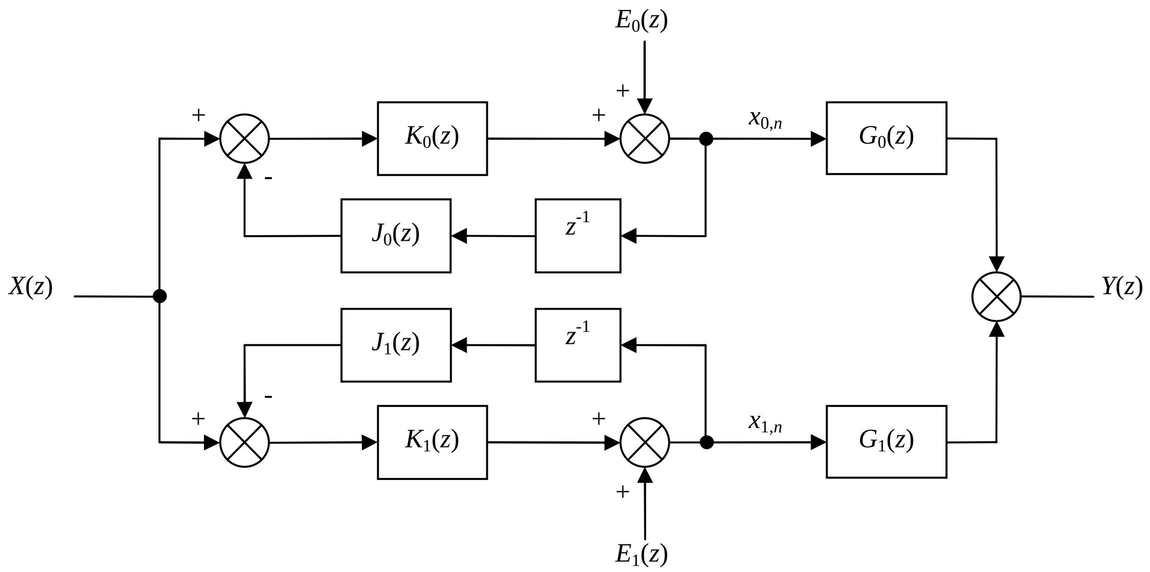


**Fig. 4:** *Dual channel filter bank composed of sigma-delta modulator*

The first two elements in the relation express the passage of the input signal through the decomposition and reconstruction filter bank. The third and fourth elements express the quantization noise which appears in individual quantizers. If these elements are minimized then the transfer of quantization noise decreases and quantizers with a lower number of quantization layers may be used to achieve the same level of quantization noise after the reconstruction filter bank. Decrement of number of quantization layers is in ones order.

## 5 CONSLUSION

The usage of sigma-delta modulators in filter bank is another possibility of their realization. Its advantage is in non-uniform inserting of quantization noise over whole spectrum of subbands signals. In compare with realization via simple FIR filters the realization via sigma-delta modulation is more complexity. The usage of this filter bank structure is in lossy data compression. The continuation of this work will be design of method how compute the individual transfer functions for chosen impulse response.

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