# THE COMPARISON OF STEEPEST DESCENT AND NEWTON'S METHOD USED FOR FREQUENCY OFFSET ELIMINATION

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## ABSTRACT

This contribution is focused on a carrier frequency offset elimination problem. The carrier frequencies of both the transmitted modulated signal and the local receiver oscillator may differ due to many known phenomena. However, the carrier waveform with an accurate frequency of received modulated signal is needed to perform a correct demodulation. Thus the carrier frequency offset is needed to be estimated and eliminated. Two optimization methods utilized for the indirect frequency estimation are compared in this paper. The steepest descent method and Newton's method are implemented in Matlab and simulated. The time dependencies of phase offset estimates and recovered message signal are the results of simulation.

## 1 INTRODUCTION

The three main channel parameters required by the most receivers for the correct signal demodulation are the carrier frequency, the carrier phase (which is not needed for non-coherent demodulation), and the symbol timing of the received signal. The carrier frequency of the received signal may differ from the carrier frequency of transmitter. The main goal is to estimate the exact carrier frequency from the received signal and to synthesize new carrier frequency waveform which can be used for demodulation. Optimization methods based on the steepest descent approach can be successfully utilized. Another optimization approach used for carrier frequency estimation is examined. It is Newton's method.

## 2 SYSTEM MODEL

Dual PLL (*Phase Locked Loop*) system [2] presented at the Fig. 1 is used for this research and for the Matlab implementation. This system provides the carrier phase estimation which can be used for the carrier frequency offset elimination thanks to a very close relation between the signal's frequency and its phase. The system is based on an optimization of a particular cost function which can be derived from a signal's representation in an input of a receiver [2]:

$$F(\theta)=\frac{1}{2}\,\mathrm{LPF}\{r(kT_s)\cos(4\pi f_c kT_s+2\theta)\} \tag{1}$$

where LPF symbolizes low pass filtering, $r(kT_s)$ symbolizes the received signal, $T_s$ is a sampling period, $f_c$ and $\theta$ are the carrier frequency and phase, respectively.

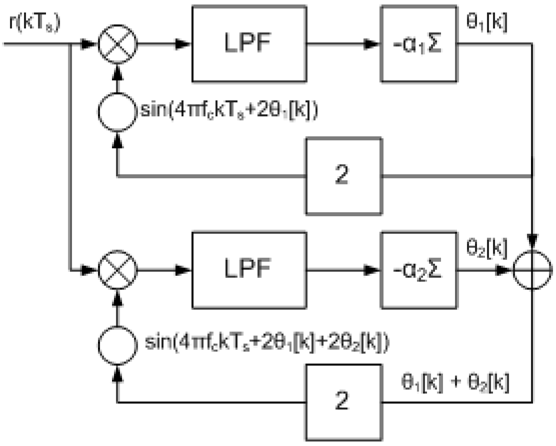


**Fig. 1:** *Dual PLL system used in model.*

The gradient method called 'Steepest Descent' is suitable and can be applied to solve this carrier offset estimation problem. The $F(\theta)$ cost function needs to be maximized while the derivation with respect to $\theta$ is applied. The derivation leads to the adaptive elements which are described by following formulas:

$$\begin{aligned}\theta_1[k+1]&=\theta_1[k]-\alpha_1\mathrm{LPF}\{r(kT_s)\sin(4\pi f_c kT_s+2\theta_1[k])\}\\ \theta_2[k+1]&=\theta_2[k]-\alpha_2\mathrm{LPF}\{r(kT_s)\sin(4\pi f_c kT_s+2\theta_1[k]+2\theta_2[k])\}\end{aligned} \tag{2}$$

where $\theta_1$ and $\theta_2$ are the estimated phase elements and $\alpha_1$ and $\alpha_2$ are the iterative stepsizes.

The second examined approach is called 'Newton's Method'. One loop is utilized for this method and the adaptive element is derived by applying first-order and second-order derivation to the cost function (1). The derivations are applied with respect to $\theta$. According to the mentioned mathematical modifications the adaptive element can be formed as:

$$\theta_3[k+1]=\theta_3[k]-\alpha_3\frac{\mathrm{LPF}\{r(kT_s)\sin(4\pi f_c kT_s+2\theta_3[k])\}}{\mathrm{LPF}\{r(kT_s)\cos(4\pi f_c kT_s+2\theta_3[k])\}} \tag{3}$$

where $\theta_3$ is the estimated phase element and $\alpha_3$ is the iterative stepsize.

## 3 IMPLEMENTATION AND SIMULATION

The equations (2) above are suitable for Matlab implementation of the steepest descent approach. They create a core of a system simulating receiver performance. The simulation model processes an $N$-level square binary message signal which is modulated using ASK modulation and particular carrier frequency. The receiver section pre-processes the received signal by squaring and filtering with a bandpass filter with a centre frequency of $2f_c$. The carrier frequency of the local oscillator is set to be different from the carrier frequency of the received signal to simulate a frequency offset. The ratio between the carrier frequency of the local oscillator and the received signal is set to 1.001. The 'unknown' received signal phase is set to -2 rad and the iterative stepsizes $\alpha_1$ and $\alpha_2$ to $5.10^{-6}$ and $5.10^{-9}$, respectively. The initial phase offsets $\theta_1(0)$ and $\theta_2(0)$ are set to 0. Fig. 2 shows the estimated phase offsets $\theta_1$ and $\theta_2$ which results in a frequency offset elimination. These estimated phase offsets are used

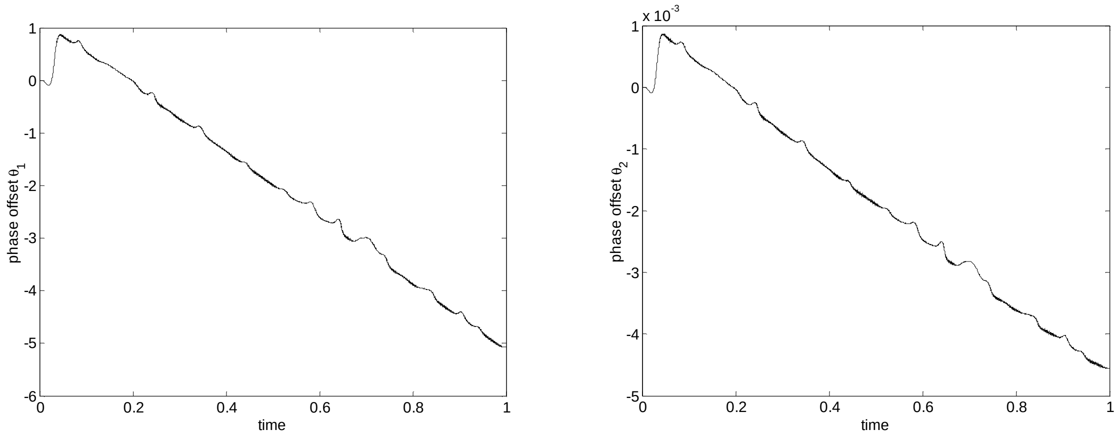


**Fig. 2:** *Phase offset $\theta_1$ (left) of the top loop and $\theta_2$ (right) of the bottom loop.*

in following operation for synthesis of a carrier frequency used in demodulation process. The stepsizes are chosen with respect to get the best results.

Fig. 3 displays the carrier frequency offset of new synthesized carrier in the receiver part of simulated communication system compared to the carrier waveform used for modulation in a pre-processor part.

Fig. 4 shows a comparison of the original message signal and the recovered message signal when the synthesized carrier is applied. A slight time shift is observable.

The equation (3) is implemented in Matlab to demonstrate Newton's method approach used for estimation of phase elements to eliminate frequency carrier offset. The pre-processor generates the same signal waveform with the same parameters as previously. The simulation engine performs the mathematical operation according to iterative equation (3). In this case, the initial phase offset $\theta_3(0)$ is set to1 rad and the stepsize $\alpha_3$ to $5.10^{-6}$. The initial phase offset and the stepsize are chosen with respect to get the best results.

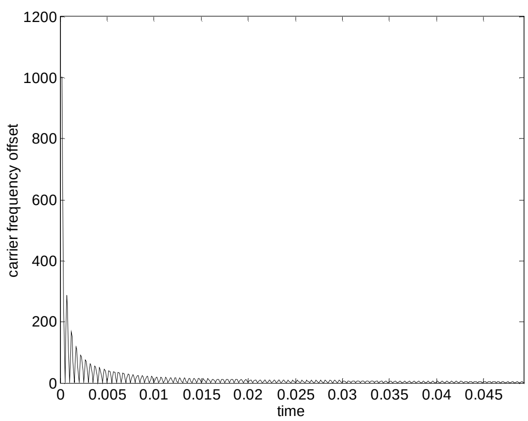


**Fig. 3:** *Frequency carrier offset (s. d.).*

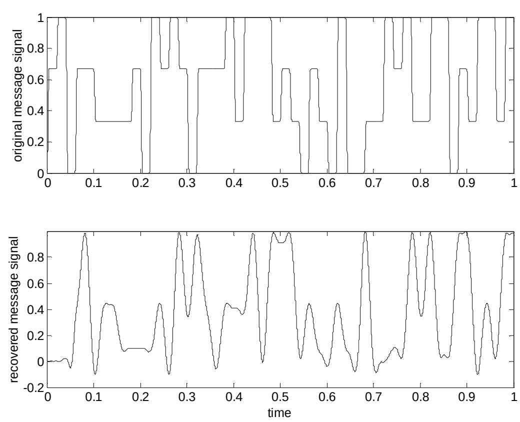


**Fig. 4:** *Original (top), recovered (bottom) signal.*

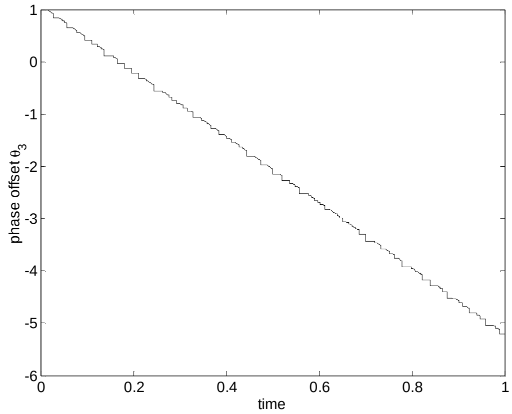


**Fig. 5:** *Phase offset $\theta_3$.*

The time dependency of estimated phase offset $\theta_3$ is presented at Fig. 5. The algorithm strictly keeps the descending trend which is proportional to the difference between the receiver's synthesized carrier and the carrier of received signal.

Fig. 6 represents the time dependency of carrier frequency offset of carrier synthesized using Newton's method. Fig. 7 again displays a comparison of the original message signal and the recovered message signal when the synthesized carrier is applied. When comparing both waveforms recovered by steepest descent method and Newton's method they appear to be very similar. The small differences can be noticed mainly in the initial time period when both of the algorithms estimate the phase offset inaccurately.

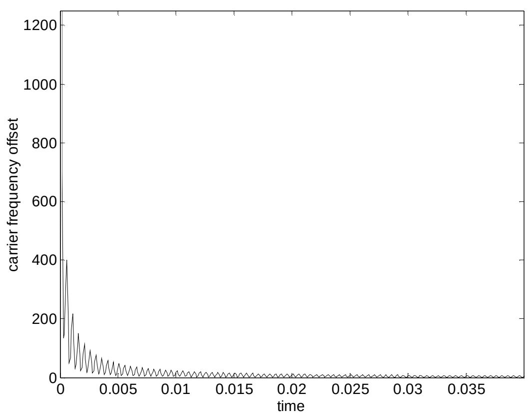


**Fig. 6:** *Frequency carrier offset (N. m.).*

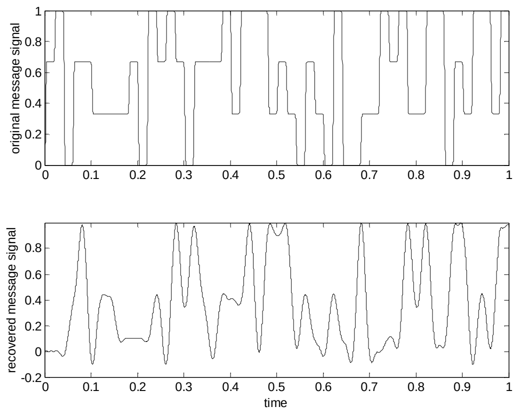


**Fig. 7:** *Original (top), recovered (bottom) signal.*

## 4 SUMMARY

The examined PLL system is very reliable and provides accurate synthesis of both the carrier frequency and phase. The main disadvantage is its sensitivity to initial setting and the setting of stepsizes $\alpha_1$, $\alpha_2$ and $\alpha_3$. High stepsize values lead to inaccurate phase estimation and incorrect demodulation. Reversely, extremely small values are inefficient and slow down the estimation process because more iterations are needed to reach the accurate value of the phase offset. It is find out that for the rough phase estimation the top PLL is sufficient when utilizing the steepest descent approach. Newton's method in comparison with steepest descent method is even much more sensitive to initial setting of initial phase offset and stepsize. Reversely, steepest descent method accepts more significant changes in both the received signal's frequency and phase. It is more flexible and thus can produce carrier frequency waveform with accurate parameters. If the initial parameters are chosen unthoughtfully for Newton's method the system is not able to estimate the phase offset accurately. The system based on steepest descent approach performs better over the environments with significant parameter changes.

## ACKNOWLEDGEMENT

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