ANALYSIS OF FREQUENCY SELECTIVE SURFACES USING SPECTRAL DOMAIN METHOD

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ABSTRACT

Spectral domain moment method (SDMM) is widely used for the analysis of frequency selective surfaces (FSS). Applying SDMM to the analysis of FSS, we need to know current distribution on conductive elements of FSS. Currents are expressed as a linear combination of basis functions multiplied by unknown coefficients. A wise choice of basis functions can reduce the computation time considerably. The current distribution is analyzed by the MATLAB program using two sets of basis functions, and in ANSOFT Designer. Results of analyses are compared.

1 INTRODUCTION

SDMM is based on the integral form of Maxwell's equations. Frequency characteristics of FSS are determined by the current distribution on the conductive elements [1].

The incident field E^{inc} induces currents on the screen, which radiate the scattered field Escat. Expressing this scattered field and considering boundary condition on the metallic portions of the screen, we get a usual form of the electric field integral equations (EFIE) [3]

$$\vec{E}^{inc}(\vec{r}) = j\omega\mu\left\{\vec{A}(\vec{r}) + \frac{1}{k^2}\vec{\nabla}\left[\vec{\nabla}\cdot\vec{A}(\vec{r})\right]\right\} \tag{1}$$

where k is wavenumber, $\vec{r}$ is a position vector, $\vec{A}$ is a vector potential, ω denotes angular frequency and μ permeability of the surroundings. Vector potential is the function of current density and is the only unknown in EIFE [1].

We assume that the structure is infinite in extent. Due to the periodicity of conductive elements on the FSS we can express current density in *xy*-plane using Fourier series. Substituting all the necessary quantities in the spatial domain to the EIFE, we get the electric field equation in the spectral domain, which is the basic equation for the numerical analysis. Current density in the spatial domain in this equation is the only unknown [1].

The unknown current distribution is approximated by the linear combination of basis functions and unknown coefficients [1].

Basis functions have to agree with boundary conditions for currents induced on the screen. Boundary conditions for induced currents expresses that currents can not run out of the FSS screen [1].

The approximation error is respected by the residual function (residuum). This residual function is minimized by Galerkin‘s method, which produces a solution corresponding to the electromagnetic field of minimal energy. After implementing these operations, the necessary number of linear algebraic equations for evaluation of unknown coefficients is obtained [1].

2 NUMERICAL SOLUTION

The rectangular planar array is analyzed. The conductive element is of the dimensions a = 13 mm, b = 1.5 mm, the cell is of the dimensions A = 15 mm, B = 7.5 mm.

For the numerical solution, some assumptions have to be done. First, the structure is infinite in extent. Second, the electric conductivity of metallic parts is perfect. Third, the FSS is fabricated from such dielectrics, which properties are identical with the surroundings. Last, the incident radiation is harmonic electromagnetic plane wave.

An often-used basis set for rectangular plate geometry is the set of TE and TM wave-guide modes [1]:

$$\psi_{pq}^{TE}(x,y)=\left[\frac{p\pi}{a}\sin(\frac{p\pi}{a}x)\cos(\frac{q\pi}{b}y)\vec{x}_0+\frac{q\pi}{b}\cos(\frac{p\pi}{a}x)\sin(\frac{q\pi}{b}y)\vec{y}_0\right]e^{j(\alpha_0 x+\beta_0 y)} \quad (2a)$$

$$\psi_{pq}^{TM}(x,y)=\left[\frac{q\pi}{b}\sin(\frac{p\pi}{a}x)\cos(\frac{q\pi}{b}y)\vec{x}_0-\frac{p\pi}{a}\cos(\frac{p\pi}{a}x)\sin(\frac{q\pi}{b}y)\vec{y}_0\right]e^{j(\alpha_0 x+\beta_0 y)} \quad (2b)$$

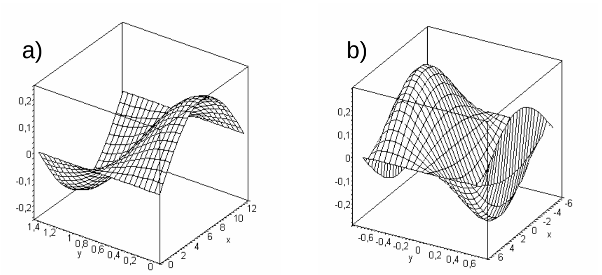

Fig. 1: *Basis functions: a) the harmonic basis function ψ_{11}^{TE}(x, y) for approximating x– component of the current density vector (a = 13 mm, b = 1.5 mm); the basis function $\psi_{13}^{(1)}$(x, y) for approximating x – component of the current density vector of (a = 13 mm, b = 1.5 mm).*

Here, a and b are the dimensions of conductive element, p and q are indices of the basic functions, α_0 and β_0 are the basic spatial frequencies.

The harmonic basis function ψ_{11}^{TE}(x, y) for approximating the x -component of the current density vector of is depicted in Fig. 1a.

Fourier transforms of basis functions are substituted to the electric field equation in the spectral domain, and unknown coefficients are evaluated. These coefficients are applied for expressing the current distribution on conductive elements of FSS.

The second type of a usable basis set is formulated by harmonic functions and Chebyshev polynomials:

$$\psi_{pq}^{(1)}(x,y)=\sin\left[\frac{q\pi}{a}(x+\frac{a}{2})\right]\cdot\frac{T_p(\frac{2y}{b})}{\left[1-\left(\frac{2y}{b}\right)^2\right]^{0.5}}\vec{x}_0+\frac{T_q(\frac{2x}{a})}{\left[1-\left(\frac{2x}{a}\right)^2\right]^{0.5}}\sin\left[\frac{p\pi}{b}(y+\frac{b}{2})\right]\vec{y}_0, \quad (3a)$$

$$\psi_{pq}^{(1)}(x,y)=\sin\left[\frac{q\pi}{a}(x+\frac{a}{2})\right]\cdot\frac{T_p(\frac{2y}{b})}{\left[1-\left(\frac{2y}{b}\right)^2\right]^{0.5}}\vec{x}_0-\frac{T_q(\frac{2x}{a})}{\left[1-\left(\frac{2x}{a}\right)^2\right]^{0.5}}\sin\left[\frac{p\pi}{b}(y+\frac{b}{2})\right]\vec{y}_0\;, \qquad (3b)$$

where T_i is the ith-order Chebyshev function of the first kind.

Fourier transforms of these functions are:

$$\begin{aligned}\psi_{pq}^{(1)}(\alpha_m,\beta_n)&=c^{'}\left\{\sin c\left(\frac{q\pi}{2}+m\pi\frac{a}{A}\right)+(-1)^{q-1}\sin c\left(\frac{q\pi}{2}-m\pi\frac{a}{A}\right)\right\}\cdot J_p\left(n\pi\frac{b}{B}\right)+\,,\\&+c^{''}J_q(m\pi\frac{a}{A})\left\{\sin c\left(\frac{p\pi}{2}+n\pi\frac{b}{B}\right)+(-1)^{p-1}\sin c\left(\frac{p\pi}{2}-n\pi\frac{b}{B}\right)\right\}\end{aligned} \qquad (4a)$$

$$\begin{aligned}\psi_{pq}^{(2)}(\alpha_m,\beta_n)&=c^{'}\left\{\sin c\left(\frac{q\pi}{2}+m\pi\frac{a}{A}\right)+(-1)^{q-1}\sin c\left(\frac{q\pi}{2}-m\pi\frac{a}{A}\right)\right\}\cdot J_q\left(n\pi\frac{b}{B}\right)-\;.\\&-c^{''}J_q(m\pi\frac{a}{A})\left\{\sin c\left(\frac{p\pi}{2}+n\pi\frac{b}{B}\right)+(-1)^{p-1}\sin c\left(\frac{p\pi}{2}-n\pi\frac{b}{B}\right)\right\}\end{aligned} \qquad (4b)$$

Here, A and B are the dimensions of the cell, a and b are the dimensions of conductive element, p and q are indices of the basic functions, J_p and J_q are qth- and sth-order Bessel functions of the first kind, respectively.

3 CONCLUSION

Current distributions for two types of basis functions calculated by Matlab are shown in Fig. 2a, b. Magnitude of marginal currents was increased with respect to reality. Current

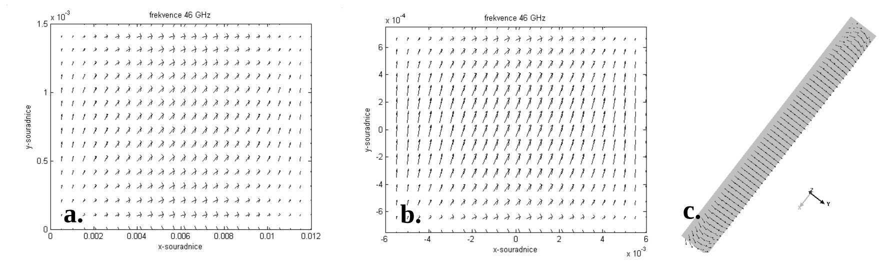

Fig. 2 : *Current distribution on conductive element of FSS*

a) *Currents approximated by harmonic basis functions*
b) *Currents approximated by combination of harmonic functions and Chebyshev polynomial*
c) *Current distribution calculated by ANSOFT Designer*

distribution calculated by ANSOFT Designer is in Fig. 2c. Frequency characteristics of analyzed FSS computed from current distributions obtained by basis functions are almost identical. Reflection coefficient is maximal on f = 14.7 GHz. Reflection coefficient computed in ANSOFT Designer has a peak on f = 13.5 GHz.

REFERENCES

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