STATE–SPACE CONTROL OF SYSTEMS WITH ELECTRICAL DRIVES

Ing. Roman KOSTKA, Doctoral Degree Programme (1)
Dept. of Power Electrical and Electronic Engineering, FEEC, BUT
E-mail: xkostk03@stud.feec.vutbr.cz

Supervised by: Prof. Jiří Skalický

ABSTRACT

In the article there is shortly introduced state-space control of the linear systems. State-space control can be also applied for feedback motion control of non-linear robotic, mechatronic and dynamic mechanical systems. In the first part of the article it is shortly resumed a state-space control theory. Practical application is shown in the second part of the article.

1 INTRODUCTION

State-space control theory is significantly applied in the control of electric drives in industry. It enables exact proposal of controllers, which are used in linear and non-linear systems. It is especially significant for the expansion of AC-motors at present. State-space control methods are acceptable for PC computation. The controllers are nearly always realized like microcomputers integrated in converters.

2 THEORETICAL PART

Electrical drive is generally introduced as a system, on which it apply p input quantities $u_1 \dots u_p$ and r disturbance quantities $z_1 \dots z_r$. The state of the system is characterized by n state variables $x_1 \dots x_n$. And there is q output quantities $y_1 \dots y_q$.

Linear, time-invariant and continuous system is described by the system of linear differential equations, with constant-coefficients. These equations can be rewritten as state equations in form:

$$\begin{aligned} \mathbf{x}' &= \mathbf{Ax} + \mathbf{Bu} + \mathbf{Ez} \\ \mathbf{y} &= \mathbf{Cx} \end{aligned} \tag{1}$$

Where:	**A**	system matrix	**y**	vector of output variables
	B	input matrix	**u**	vector of input variables

C	output matrix	**z**	vector of disturbancies
E	disturbance matrix	**x**	vector of state variables

We must verify controllability and observability of the system before designing the controller. If the system is not controllable and observable it must be appropriately set up[1].

Values of state variables in defined time will be obtained as a result of the solution of state equations.

For the system described by state equations will be proposed a control, which ensure stability, desired static accuracy and dynamic of control circuit in the action of the command variable and the disturbance variable.

State space control supposes complete system information. It means that every state variable is measurable. When this condition is not possible to be realized, control system must be realized with a state observer. Signals corresponded with immeasurable state variable are obtained from the state observer.

Disturbances, in electric drives primarily it is a load torque, can largely affect on drive's characteristics. Therefore the most used method of control is the addition of an integration component into the controller and complementation the state observer with a disturbance observer.

The eigenvalues of a matrix **A** gives dynamic characteristics of the system and they are the poles of the system, too. The location of poles in a complex plane designates stability, eigenfrequency and damping of the system.

Designing of the state feedback regulator consists in the new pole-placement of the closed-loop. It changes the dynamic behaviour of the closed-loop. State-space controller makes linear feedback from all state variables.

$$\mathbf{x}' = \mathbf{Ax} + \mathbf{B}(-\mathbf{Rx} + \mathrm{w}) + \mathbf{Ez} = (\mathbf{A} - \mathbf{BR})\mathbf{x} + \mathbf{B}\mathrm{w} + \mathbf{Ez} \qquad (2)$$

Where $(\mathbf{A} - \mathbf{BR})$ is a new system matrix with closed feedback loop. Eigenvalues of new system matrix are roots of the characteristic equation:

$$\det |p\mathbf{I} - \mathbf{A} + \mathbf{BR}| = 0 \qquad (3)$$

We choose new poles of the closed loop:

$$\prod_{i=1}^{n}(p - p_i) = p^n + a_{n-1}p^{n-1} + \ldots + a_1 p + a_0 \qquad (4)$$

To determine **R** we compare equations (3) a (4).

3 PRACTICAL SOLUTION

Design of the state-space controller is suitable for more complicated dynamics systems.

This system can be represented by an electrical drive of a milling stand. It presents a four-masses (two DC motors-masses, gearbox-mass, load-mass) system with a flexible connections (see fig.1). The control block represents both electrical and control parts of the drive. We assume that the system is linear.

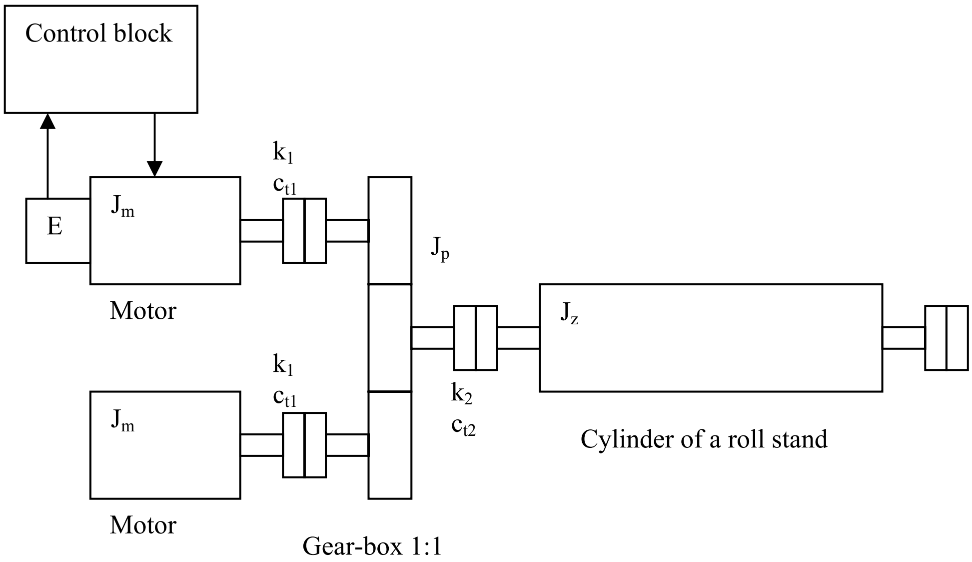

Fig. 1: *The model of a system*

The system on fig.1 is described by these system equations:

$$
\begin{aligned}
\frac{d\omega_{m1}}{dt} &= \frac{1}{J_{m1}}\left(-k_1\omega_{m1} + k_1\omega_p - c_{t1}\varphi_{m1} + c_{t1}\varphi_p + M_{m1}\right)\\
\frac{d\omega_{m2}}{dt} &= \frac{1}{J_{m2}}\left(-k_1\omega_{m2} + k_1\omega_p - c_{t1}\varphi_{m2} + c_{t1}\varphi_p + M_{m2}\right)\\
\frac{d\omega_p}{dt} &= \frac{1}{J_p}\left[k_1\omega_{m1} + k_1\omega_{m2} - (2k_1 + k_2)\omega_p + k_2\omega_z + c_{t1}\varphi_{m1} + c_{t1}\varphi_{m2} - (2c_{t1} + c_{t2})\varphi_p + c_{t2}\varphi_z\right]\\
\frac{d\omega_z}{dt} &= \frac{1}{J_z}\left(k_2\omega_p - k_2\omega_z + c_{t2}\varphi_p - c_{t2}\varphi_z\right)\\
\varphi'_{m1} &= \omega_{m1}\\
\varphi'_{m2} &= \omega_{m2}\\
\varphi'_p &= \omega_p\\
\varphi'_z &= \omega_z
\end{aligned}
\tag{5}
$$

As inputs we consider motor-torques M_m, the output is the angular speed of the load ω_z. Parameters J_m, J_p, J_z are moments of inertia of the motor, of the gear-box and of the load, ω_m, ω_p, ω_z are angular speeds of the motor, of the gear-box and of the load, φ_m, φ_p, φ_z are shaft-angles of the motor, of the gear-box and of the load, c_t is an elasticity-coefficient and k is a damping factor of the elastic mechanic coupling. The load is a cylinder of the roll stand.

To design the state-feedback controller it is used the method of the pole-placement of the characteristic equation of the closed-loop. To compensate of disturbance there is added an integration part into the controller.

The system equations (5) we rewrite as a state equations. Then we can design a block diagram of a control system. The block diagram of the control system is in fig. 2.

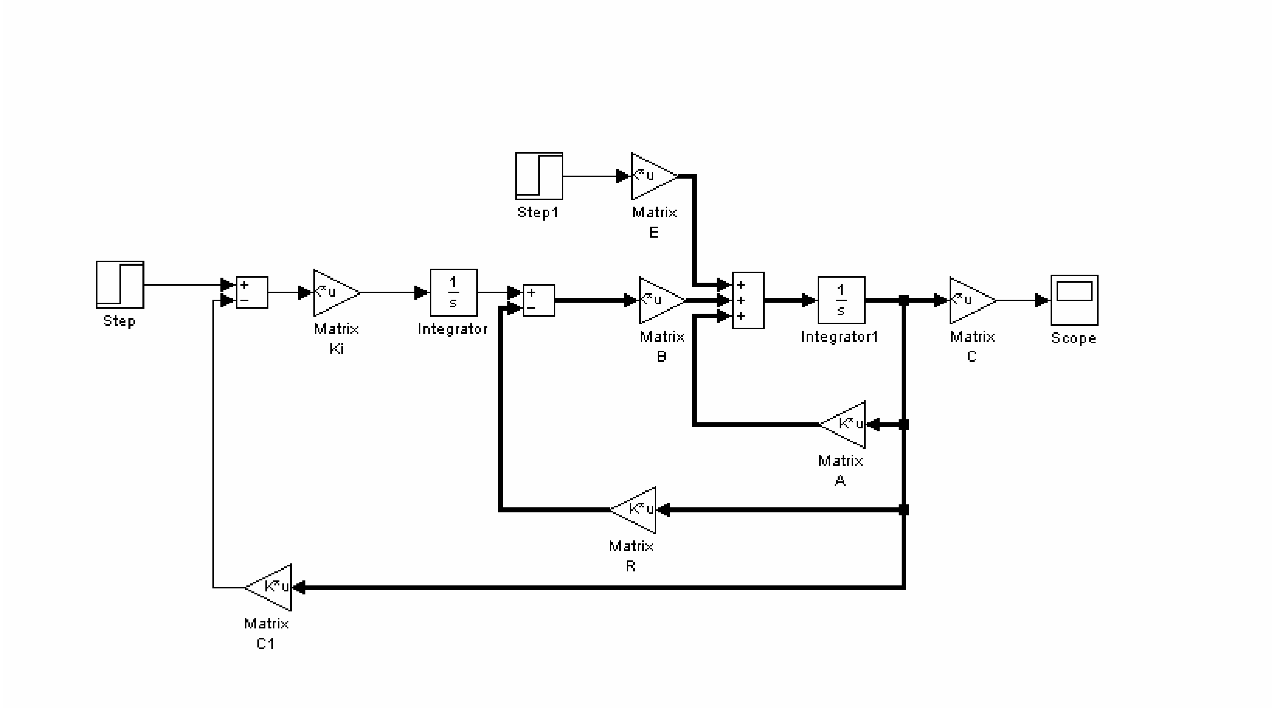

Fig. 2: *The block diagram*

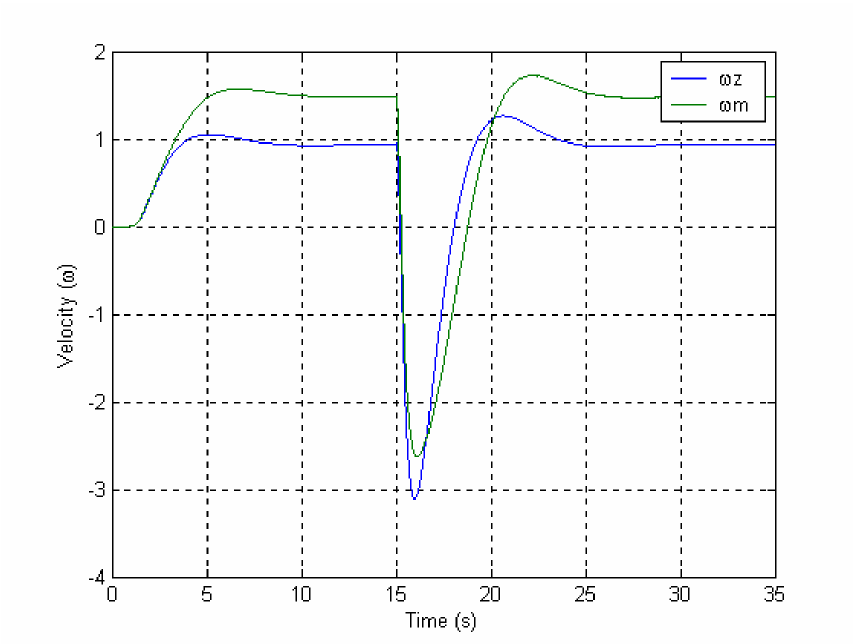

Fig. 3: *Step response*

There is step response of the system in fig.3. We can see curves of state variables ω_m, ω_z. There is disturbance (load torque) in time 15 second.

The block diagram (fig. 2) and step response (fig. 3) are created in Matlab-Simulink.

4 CONCLUSION

The article discuss about a state-space control of linear systems. The simulation proves that the design of the state-space controller is suitable for more complicated dynamics systems. The state-space control is a suitable tool how to control non-linear dynamic system as well.

ACKNOWLEDGEMENT

The paper has been prepared as a part of the solution of project: MSM 0021630516.

REFERENCES

[1] Zboray, L., Ďurovský, F.: Stavové riadenie elektrických pohonov , FEI Košice, 1995

[2] Skalický, J.: Teorie řízení, VUT v Brně, 2002