COVERAGE FACTOR APPROXIMATION IN EXPANDED UNCERTAINTY EVALUATION

Ing. Petr FRANK, Doctoral Degree Programme (3)
Dept. of Power Electrical and Electronic Engineering, FEEC, BUT
E-mail: frank@feec.vutbr.cz

Supervised by: Prof. Karel Hruška

ABSTRACT

This paper deals with the usability of an approximation in extended uncertainty evaluation. Improper approximation can cause that the expanded uncertainty won't be at the required confidence level. But some approximation is always needed in practical uncertainty evaluation. The Scope of this paper is to analyze acceptability of common used approximation in coverage factor evaluation.

1 INTRODUCTION

Expanded uncertainty represents an interval that contains true value on the required confidence level and it is defined in document "Guide to the Expression of Uncertainty in Measurement" [1]. In practice there are frequently used approximations of the uncertainty evaluation, because it is difficult to use theoretical methods in practical measurement. These approximations can cause improper uncertainty evaluation. Task of this paper is to analyze common used coverage factor approximation in case of direct measurement.

2 EXPANDED UNCERTAINTY

There are many sources that describe uncertainty evaluation, e.g. [2]. Problem is, that most of the sources don't enter details of expanded uncertainty evaluating. Some sources (e.g. [3]) describe expanded uncertainty only as combined uncertainty multiplied by coverage factor without any other details. Coverage factor is often recommended to be $k = 2$ for confidence level 95 % and $k = 3$ for confidence level 99,7 % (or 99 %, e.g. [4]). Majority of the sources (e.g. [5]) goes deeper and describes evaluating of coverage factor with help of Student's distribution and effective degrees of freedom. It is recommended, according to the GUM [1], to use Welch-Satterthwaite approximation formula (3) as a possible method for calculating of effective degrees of freedom.

As mentioned above, the expanded uncertainty is calculated as combined uncertainty multiplied by so-called coverage factor, so that

$$U = k \, . \, u_c, \tag{1}$$

where U is expanded uncertainty, k is coverage factor and u_c is standard combined uncertainty. Problem is how to compute proper coverage factor. Task of a coverage factor is to expand interval that belongs to the combined uncertainty so, that final interval will contain true value with required confidence level. In case of normal distribution and n independent observation it is possible to set coverage factor from the t-distribution (Student's) table. In compliance with [2] will then degrees of freedom be $\nu = n - 1$. In Student's table are tabulated values for various degrees of freedom and confidence level, so we could write

$$U = k \cdot u_c = t_p(\nu) \cdot u_c, \qquad (2)$$

where $t_p(\nu)$ is t-distribution coefficient and ν is number of degrees of freedom. Degrees of freedom related to uncertainty estimation depend on amount of information that was used for the estimation. If the amount of information grows then degrees of freedom grows too. When uncertainty consist of more than just one part type A with $\nu = n - 1$ degrees of freedom, then it is necessary to use Welch-Satterthwaite formula for effective degrees of freedom

$$\nu_{ef} = \frac{u_c^4}{\sum_{i=1}^{N} \frac{\left(\frac{\partial f}{\partial x_i}\right)^4 u_i^4}{\nu_i}}, \qquad (3)$$

where u_i are all particular uncertainties, that participated on the standard combined uncertainty u_c, and ν_i are degrees of freedom of these particular uncertainties. There is also employed the partial derivation of the output quantity by input quantities x_i, and ν_i are degrees of freedom of input uncertainties. For uncertainties type A are degrees of freedom $\nu_i = n(u_i) - 1$, where $n(u_i)$ is a data amount used for uncertainty u_i evaluation. For type B uncertainties is the situation more complicated.

3 DEGREES OF FREEDOM FOR UNCERTAINTY TYPE B ESTIMATION

Uncertainties type B are evaluated, for example, from test equipment accuracy, from expert's estimation and other uncertainty sources. In the most of practical measurements is uncertainty type evaluated from TME accuracy. The problem isn't only in the uncertainty itself, which can be calculated e.g. according to [2], but there is also problem in its degrees of freedom. As mentioned above, finite degrees of freedom are related with situation where at least one particular uncertainty isn't known exactly. Often used approximation is to have infinite degrees of freedom for type B uncertainty. But this may not be true, because in case of expert's estimation we aren't sure about it and in the fact, we are more confident about uncertainty type A. So when have we to compute degrees of freedom for uncertainty type B and when not? It depends on how will the change of type B uncertainty and change of its degrees of freedom affect the Welch-Satterthwaite formula and therefore also combined uncertainty.

4 APPROXIMATION OF COVERAGE FACTOR

According to equation (2) is coverage factor evaluated from Student's distribution table.

At Fig. 1 is shown dependence of coverage factor (or if you like coefficient t_p) on degrees of freedom and on confidence level. There are degrees of freedom from 10 to 35 and there are also shown values for infinite degrees of freedom.

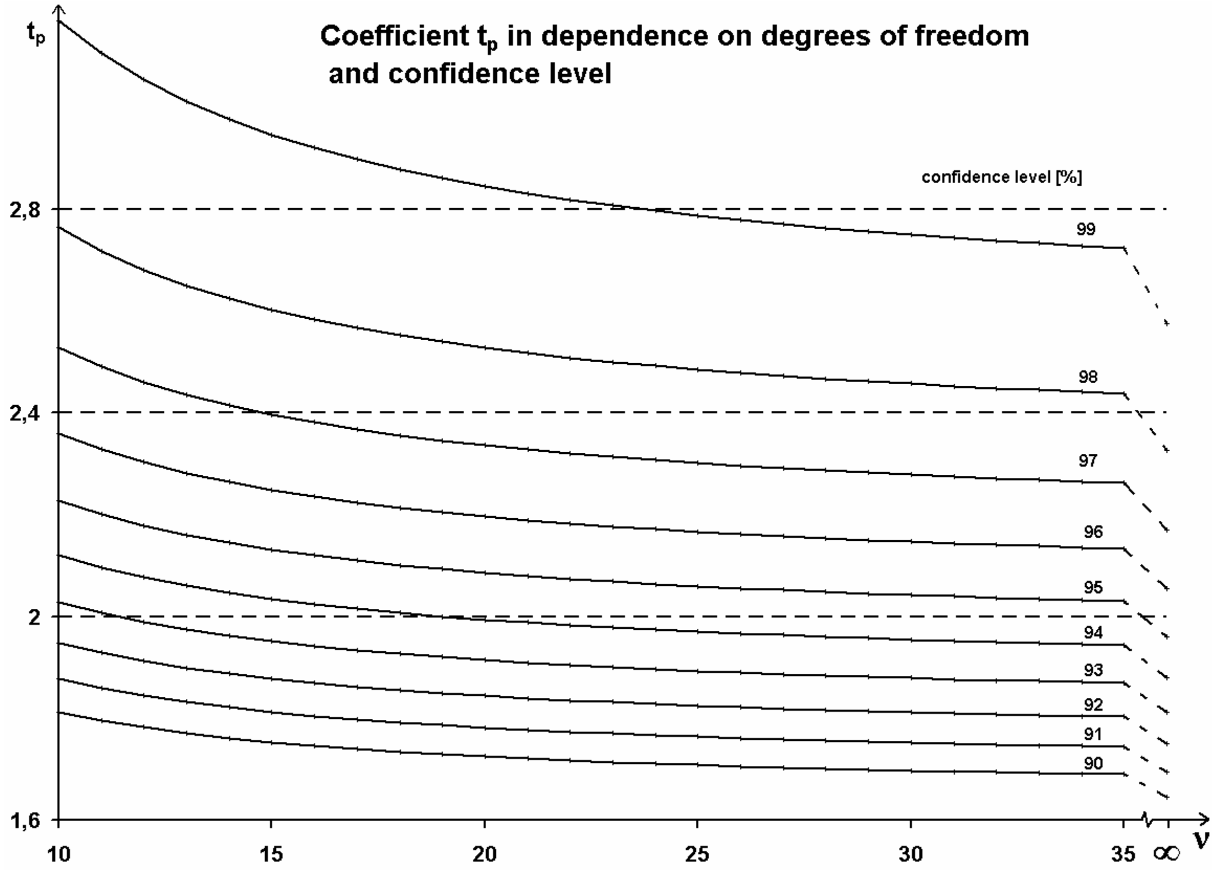

Fig. 1: *Coefficient t_p in dependence on degrees of freedom and confidence level*

The most widely used confidence levels are 95 % and 99 % or so-called 2 and 3 Sigma that are related with values 95.45 % and 99,73 %. Unfortunately, these two values are often mismatched, and with 95 % confidence level it is mean the level 95,45 %. Sometimes it isn't mistake but only approximation. Common used coverage factor $k = 2$ a $k = 3$ are in fact coefficient of Student's distribution for infinite degrees of freedom. Because of this approximation grows the expanded uncertainty and therefore grows also the like-hood that true value will lie in uncertainty interval with probability of 95 or 99 %. For us is important to examine where the line for 95 % will cross the value 2 of coverage factor. Before this crossing is not possible to use $k = 2$ for confidence level 95 %. For coverage factor 3 is situation better, because there it is no problem to achieve confidence level 99 %.

Lets focus on direct measurement, where uncertainty type B will be calculated from measurement accuracy and expanded uncertainty will be on 95 % confidence level. The question is, how high can the ratio u_B/u_A , ν_A and ν_B be? Answer is in S-W equation (3). When we extend the graph from Fig. 1 then we can see that the crossing for $k = 2$ and 95 % is at 61 effective degrees of freedom. So we need find such ratio u_B/u_A , ν_A and ν_B that will produce effective degrees of freedom equal or higher than 61. At Fig. 2 is graphic solution.

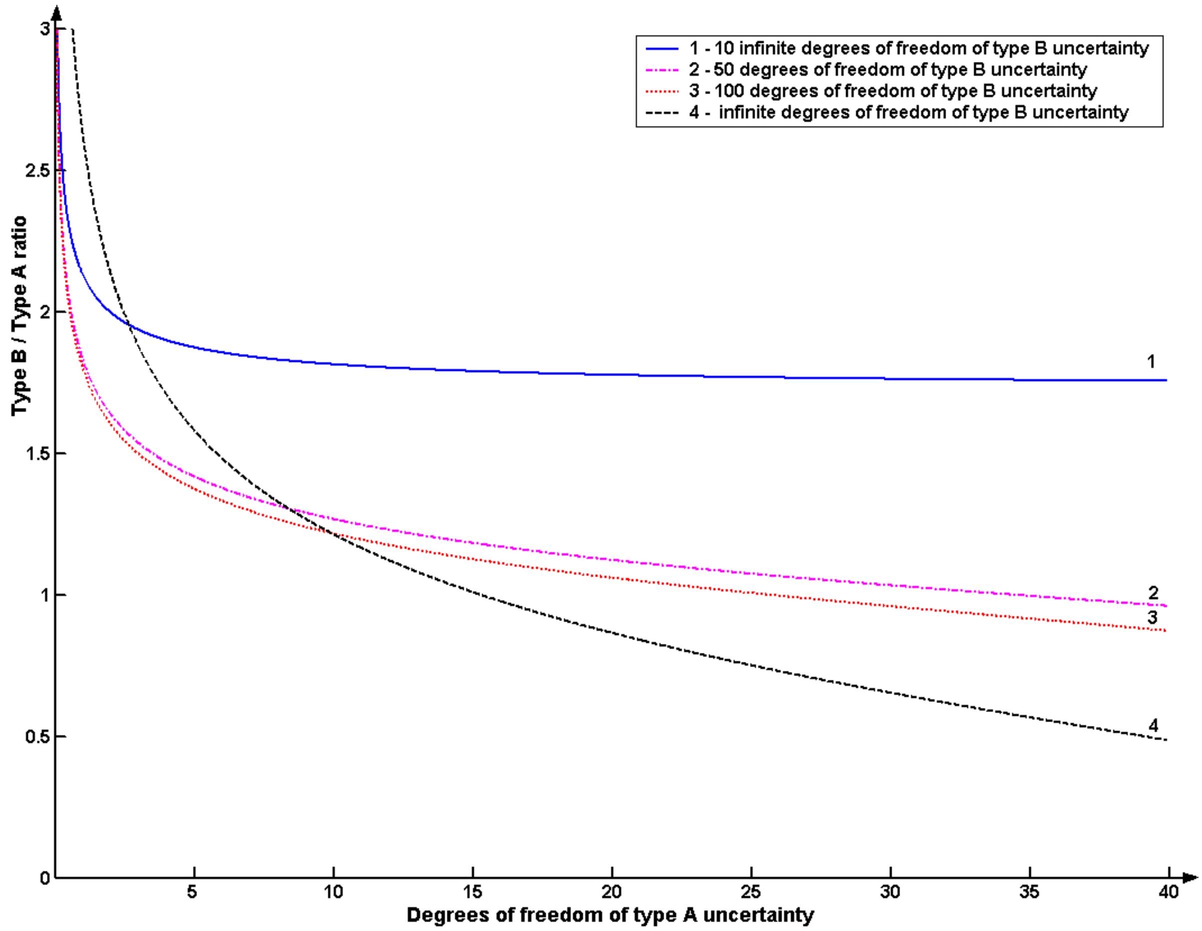

Fig. 2: *Minimal u_B/u_A ratio and ν_A for 61 effective degrees of freedom*

I used degrees of freedom of uncertainty B infinite (what is commonly used) and also equal to 10, 50 and 100 as an example of number of degrees of freedom. For 1 degree of freedom is minimum u_B/u_A ratio 2.61 and for $\nu_A = 2$ is the minimum ratio 2.16. The conclusion is that if uncertainty type A is 3 times less than uncertainty type B then there is no need to do the measurement than twice. This conclusion is nonsense, because minimal count of degrees of freedom for statistic is 2 and for measurement is absolute minimum 6. So why does S-W provide such results? In [6] it is pointed to that S-W can't be used when some degrees of freedom are too small. In my opinion it is not possible to handle with uncertainty type A calculated from 2 or 3 measurement. So we must read Fig. 2 from 5 at axis x.

5 CONCLUSION

In every case there is need to calculate uncertainty type A from sufficient count of measurement. If we suppose degrees of freedom of type uncertainty as infinite and when uncertainty type B will be at least twice as big as uncertainty type A, then it is possible to use coverage factor $k = 2$ for 95 % confidence level. If uncertainty is bigger than this then it is necessary to calculate effective degrees of freedom from the Welch-Satterthwaite formula. If we will achieve as low as possible uncertainty, than it is always better use W-S formula, because for 95% confidence level we can obtain coverage factor up to $k = 1.96$. Other question is possibility of infinite degrees of type B uncertainty presumption. But from Fig. 2 it is clear that the approximation $k = 2$ in case $u_B/u_A = 2$ is also usable with $\nu_B = 10$. So we will not make any mistake when we use approximation $k = 2$.

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