IMAGE FILTERING WITH UNDECIMATED WAVELET TRANSFORMS

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ABSTRACT

There are several methods for noise reduction. Classical techniques with median or simplified Wiener filter don't always work as we expect. Newer methods with wavelet transform are now often used in image processing. Decimated discrete wavelet transform has been used for compression as well as for filtering. But in many instances undecimated version of wavelet transform works significantly better in noise reduction phase. In this paper separable and non-separable undecimated wavelet transforms are presented.

1 INTRODUCTION

Almost every kind of data contains noise. Noise reduction is a required step for any sophisticated algorithms in computer vision and image processing. This problem has existed for a long time and yet there is no good enough solution for it. A tradeoff between the removed noise and the blurring in the image always exist. The use of wavelet transform for signal de-noising has been started in last decade. Wavelets capability to give detail spatial-frequency information is the main reason for this investigation. This property promises a possibility for better discrimination between the noise and the real data. Successful exploitation of wavelet transform might lessen the blurring effect or even overcome it completely.

There are two main types of wavelet transform with decimation and without decimation. The decimated transform is very efficient from the computational point of view. Its only drawback is that it is not translation invariant. Translations of the original signal lead to different wavelet coefficients. In order to overcome this and to get more complete characteristic of the analyzed signal the undecimated wavelet transform was proposed. The general idea behind it is that it doesn't decimate the signal. Thus it produces more precise information for the frequency localization. From the computational point of view the undecimated wavelet transform has larger storage space requirements and involves more computations [2].

These methods are suitable for additive noise reducing (classical photos) [1] and work well in cases where image is corrupted with some kind of multiplicative noise (ultrasound images) [4].

2 SEPARABLE TRANSFORM

This transform is directly derived from 1D version. Each row is filtered separately with one dimensional high-pass $G(z)$ and low-pass filter $H(z)$. Then the same process is done with columns. After one step we obtain a low-pass version of an image, and horizontal, vertical and diagonal details. In a next step, we do not downsample as in classical scheme. Instead, decomposition filters are transformed with $z \rightarrow z^2$. If n a level of decomposition then filters in each decomposition is obtained as:

$$G_1(z) \rightarrow G_2(z^2) \rightarrow G_3(z^4) \rightarrow G_n(z^{2^{(n-1)}})$$

$$H_1(z) \rightarrow H_2(z^2) \rightarrow H_3(z^4) \rightarrow H_n(z^{2^{(n-1)}})\,. \tag{1}$$

Transform $z \rightarrow z^2$ means that a null is inserted between each filter coefficient of length m. Length of a new filter is $2*m - 1$ and a pass band is half.

Threshold λ_n for wavelet coefficients in level n is calculated from:

$$\lambda_n = c \cdot \sqrt{\frac{1}{N_n}\sum d_n}\ ,\ n = 1, 2, \ldots \mathrm{k} \in \mathrm{N}, \tag{2}$$

where d is set of N coefficients, while c is a parameter for manual adjusting. Example of this method with a real photograph corrupted with a additive noise is in Fig. 1; 3 decompositions, bi-orthogonal (2,2) filter bank, hard thresholding.

Fig. 1: *Separable wavelet transform: original image – left, filtered image - right*

3 NON-SEPARABLE TRANSFORM

In this work a *quincunx* transform is used as an easiest of all real two-dimensional transforms. Decompositions are taken with 2D filters in one step instead of rows and columns separately. These filters are derived from 1D originals using McClellan transform and we obtain diamond-shape 2D filters for odd decomposition steps [3]. For even decomposition steps these filters are rotated in 45 °. Decompositions filters have to be up-sampled by two, zeros are inserted between each filter coefficient horizontally and vertically.

Threshold estimation is similar as with filtering with separable transform. It is often useful to have a parameter for manual adjusting for better results. Example of this method with an ultrasound image corrupted with a multiplicative noise is in fig. 2; 6 decompositions, bi-orthogonal (2,2) filter bank, hard thresholding.

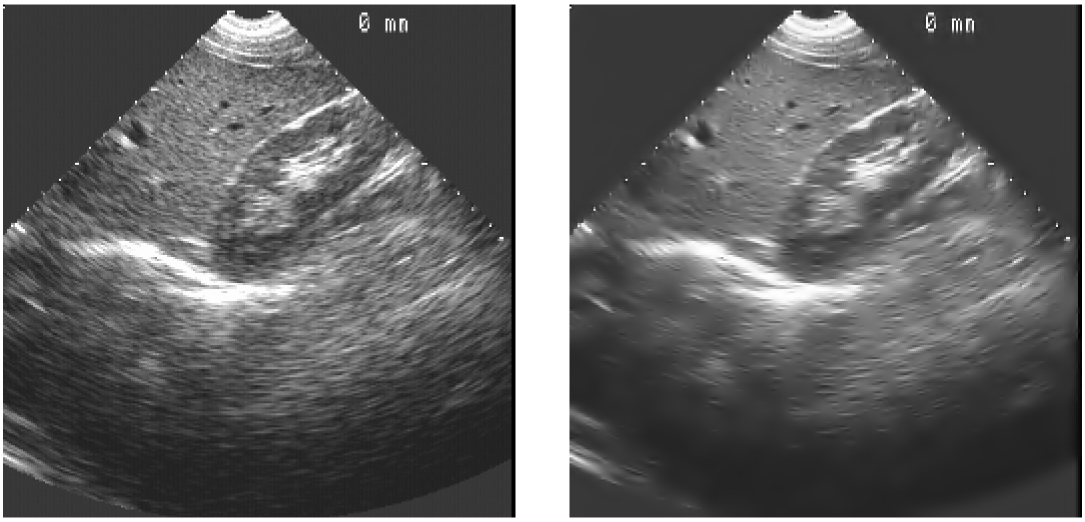

Fig. 2: *Non-separable wavelet transform: original image – left, filtered image - right*

4 DISCUSION

Both transforms are similar and it is difficult to say which one is better. When using discrete wavelet transform with down-sampling the choice of the filter bank was very important. When a method without decimation is used we can use almost any filter bank. Since images are processed it is useful to have short filters for better spatial resolution of coefficients.

So the key factor is to estimate level of noise in image. Additive noise is supposed to have the constant energy throughout the decomposition levels. Multiplicative noise is not constantly represented in separate decompositions because of its character. It depends on a signal strength and that is why it is much more difficult to find appropriate threshold for filtering. For multiplicative noise reduction non-separable transforms seems to be preferable while for additive noise in natural photos separable one is more suitable.

Both types of wavelet transform are very memory consuming because do not down-sample the image. It is often necessary to divide the whole image into several sub-images and process them separately. Because convolution is still performed on the full image and on full coefficient sets it is also quite time consuming.

Fig. 3 shows decomposition high-pass filters for both transforms. These 2D filters have the same size of 5×5 coefficients. Separable filter bank might better differentiate separate vertical and horizontal details, so separable transform might work better with photographs. In medical imaging (like ultrasound) images are not only noised but also often blurred. In this case non-separable filters with work better and noise reduction is higher.

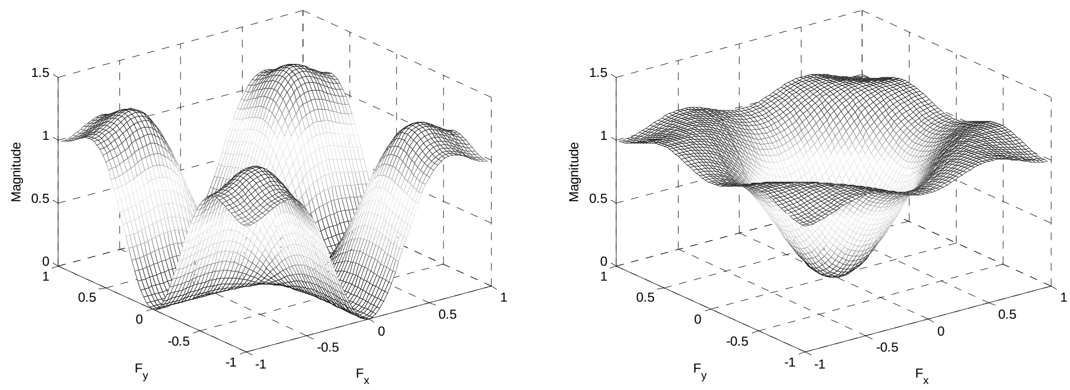

Fig. 3: *Frequency characteristics for high-pass filters for separable (left) and quincunx (right) wavelet transform*

5 CONCLUSION

Discrete wavelet transform is useful for image processing, especially for noise reduction. This type of non-linear filtering preserves details and edges very well while noise is highly suppressed. It is very difficult to say that filtered images are always better-looking that noisy originals but for further processing (like segmentation) these methods work well.

ACKNOWLEDGEMENTS

The paper has been prepared as a part of the solution of GAČR project No. 102/04/0472.

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