QUANTUM INFORMATION

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ABSTRACT

Quantum information is a rapidly developing field. An application of quantum information theory is in new technologies like ultimate safe cryptography and quantum computers. For this, there is a need to control the quantum states. In the article, there is given an introduction to the quantum information theory. We concentrate our attention how to measure quantum information. Next, the difference between classical and quantum information is discussed.

1 INTRODUCTION

When we want to obtain some information from a quantum state of the particle, we must measure this state. The measurement device is composed from two parts [2]. The first is a separator which has an arbitrary number of channels (in fact, the number isn't arbitrary but it depends on the type of the particle). The second part is a detector. It detects which channel has the particle chosen. So the measurement device is composed from a separator and a detector. If some channels are obscured, the measurement device is called filter. The ideal filter has open only one channel.

There are several sections in the article. Section 2 deals about measuring device. Mixed quantum states can be described by density operator. This is shown in section 3. The most important is section 4, where is the quantity of quantum information defined. With using a measurement device and a preparing device quantum information can be converted to classical and vice versa. This conversion is illustrated in section 5. There is also shown that this conversion is impossible without the original state. Finally, the article is summarized in section 6.

2 MEASURING DEVICE

The measuring device is described by a projection operator $\hat{P}_{\sum\limits_k b_k}$ which is seen to be a projection onto the k-dimensional subspace, defined by the normalized vectors $\left| b_k \right\rangle$ [2]

$$\hat{P}_{\sum_k b_k} = \sum_k \hat{P}_{b_k} = \sum_k |b_k\rangle\langle b_k|. \quad (1)$$

The difference between $\hat{P}_{\sum_k b_k}$ and $\hat{P}_{b_k}$ is illustrated in fig. 1. In fig. 1(a), there is the measuring device which has infinitely poor resolution. In the other words, if the particle is detected (it has got through some channel b_k), we don't know in which state $|b_k\rangle$ the particle is. The ideal measurement device is shown in fig. 1(c). Noted that b_k is called as eigenvalue and $|b_k\rangle$ as eigenstate of the operator $\hat{P}$.

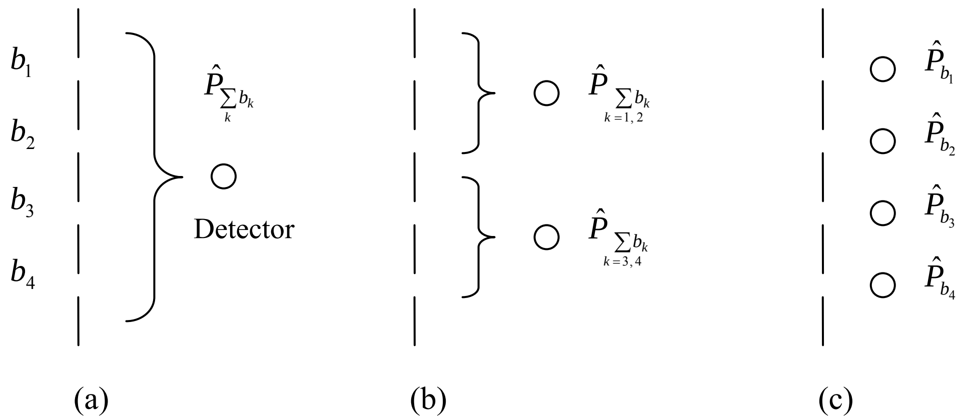

Fig. 1: *Resolution of measuring device*

3 DENSITY OPERATORS

Assume one particle in state $|\varphi\rangle$. This state will be measured. The outcome of measuring is

$$|\varphi'\rangle = \hat{P}_{\sum_k b_k} |\varphi\rangle \quad (2)$$

where $|\varphi'\rangle$ is the state of particle in the output of the measuring device. States of particles (pure or mixed) are in quantum mechanics describe by a density operator [2]

$$\hat{W} = \sum_j w_j |\varphi_j\rangle\langle\varphi_j| \quad (3)$$

where w_j is the probability that $|\varphi_j\rangle$ is chosen. In the formula (3) there is used index j, which is absolutely different of k. Index k denotes the number of eigenstates of the projection operator $\hat{P}$. The state $|\varphi_j\rangle$ since doesn't relate to $\hat{P}$ but it can be expressed as superposition of its eigenstates $|b_k\rangle$. The trace (the sum of diagonal elements) of matrix $\hat{W}$ determinates the probability of passing (detecting). Let's denote $\hat{W}'$ as the density operator after passing the filter (measuring device). For the system with infinitely poor resolution it can be written [2]

$$\hat{W}'_{ipr} = \hat{P}_{\sum_k b_k} \hat{W} \hat{P}_{\sum_k b_k} . \qquad (4)$$

For the system with ideal resolution [2]

$$\hat{W}'_{ir} = \sum_k \hat{P}_{b_k} \hat{W} \hat{P}_{b_k} . \qquad (5)$$

And for the system with finite resolution [2]

$$\hat{W}' = \sum_l \hat{P}_l \hat{W} \hat{P}_l \qquad (6)$$

where $\hat{P}_l = \hat{P}_{\sum_m b_m}$, m are channels which can not be distinguished (in fig. 2(b) these channels are 1 and 2, and the other group is composed of channels 3 and 4). The index l is a order number of groups. The relation (6) is a general formula.

4 QUANTUM INFORMATION MEASURING

Before defining the quantity of quantum information, there will be shown the expression of traces of $\hat{W}'$. For the system with infinitely poor resolution it can be written

$$\mathrm{Tr}\hat{W}'_{ipr} = \sum_n \langle b_n | \hat{P}_{\sum_k b_k} \hat{W}_{ipr} \hat{P}_{\sum_k b_k} | b_n \rangle = \sum_k \underset{\varphi\to b_k}{\mathrm{A}} \sum_k \underset{\varphi\to b_k}{\mathrm{A}^*} = \left| \sum_k \underset{\varphi\to b_k}{\mathrm{A}} \right|^2 \qquad (7)$$

where $\underset{\varphi\to b_k}{\mathrm{A}}$ is amplitude of probability that the outcome of measuring will be state $|b_k\rangle$. The probability of this event is square of the amplitude. Note, to derive relation (7), there was used the orthogonality of $|b_k\rangle$ and $W = |\varphi\rangle\langle\varphi|$ (assume only one mixed state $|\varphi\rangle$). Analogously for the system with ideal resolution

$$\mathrm{Tr}\hat{W}'_{ir} = \sum_n \langle b_n | \sum_k \hat{P}_{b_k} \hat{W}_{ir} \hat{P}_{b_k} | b_n \rangle = \sum_k \underset{\varphi\to b_k}{\mathrm{A}} \underset{\varphi\to b_k}{\mathrm{A}^*} = \sum_k \left| \underset{\varphi\to b_k}{\mathrm{A}} \right|^2 . \qquad (8)$$

And finally, for the system with finite resolution

$$\mathrm{Tr}\hat{W}' = \sum_n \langle b_n | \sum_l \hat{P}_l \hat{W} \hat{P}_l | b_n \rangle = \sum_l \left| \sum_{l,m} \underset{\varphi\to b_{l,m}}{\mathrm{A}} \right|^2 . \qquad (9)$$

If the state $|\varphi\rangle$ is measured by device with infinitely poor resolution, the output state is the same ($|\varphi'\rangle = |\varphi\rangle$). Hence, there is no information gain. If the measuring device is ideal, the maximum of information is obtained. To characterize this information gain, there is defined the quantity of quantum information

$$QI = \log_N \frac{\mathrm{Tr}W'_{ipr}}{\mathrm{Tr}W'} \qquad (10)$$

where $\hat{W}'$ is a density operator of used measuring device, which has N channels (eigenvalues). For the systems in fig. 1 there can be obtained $QI_a = 0$, $QI_b = 1/2$, $QI_c = 1$.

5 INFORMATION CONVERSION

In this section, there is presented the conversion of quantum information to classical. This is done in two ways. First, we suppose a prepared quantum state $|\varphi\rangle$ which is following measured (convert to classical information). This system is compared with the second one that contains two conversions. These systems are drawn in Fig. 2.

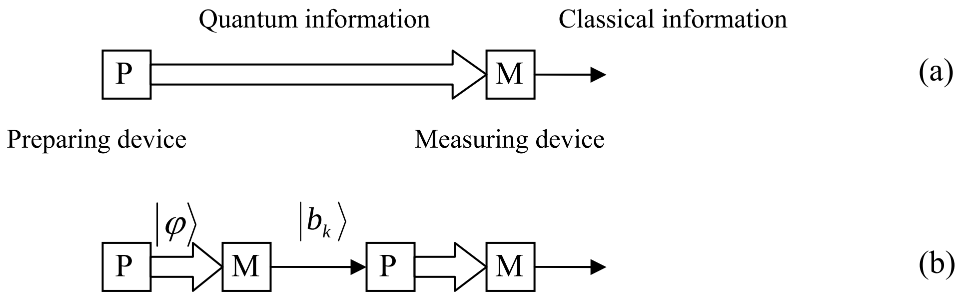

Fig. 2: *Conversion of information*

Are systems in fig. 2 equivalent? To find the answer, let's assume the initial unknown quantum state $|\varphi\rangle$. The preparing device is characterized by operator $\hat{R}$. And measuring by $\hat{P}$ (section 2). Let us denote $|\varphi'\rangle$ as the result of the measurement. Thus, the output state of the system in fig. 2(a) can be described by the formula (2). And the output state of the second system (Fig. 2(b)) satisfies

$$|\varphi'\rangle = \hat{P}\hat{R}\hat{P}|\varphi\rangle = \hat{P}|\varphi\rangle. \qquad (11)$$

By virtue of the relations (2) and (11) it can be said that both systems shown in Fig. 1 are equivalent. But the problem is how to construct the preparing device. Can it exist? It must be described by the operator

$$\hat{R} = \hat{P}^{-1} = |\varphi\rangle\langle\varphi|. \qquad (12)$$

So $\hat{R}$ projects the measured state back to the original state.

Both systems in fig. 2 are identical but there is needed the device which is represented by the operator $\hat{R}$. Have a look at the formula (12) that defines its. There is needed the original quantum state $|\varphi\rangle$! So there is a need to clone the initial state $|\varphi\rangle$. But it is impossible to perfect copy an unknown quantum state (the proof can be found in [1]).

But in fact, measuring needn't to reduce $|\varphi\rangle$ (there is no disturbance of the quantum state). So the original unknown state can be transferred although there was performed some measurement. This system requires some new components. The most important is EPR source (source which produces two particles in a singlet-spin state). This solution is presented in [3].

6 SUMMARY

In this article, we have discussed about quantum information. In the paper was defined the quantity of quantum information. Next, there was shown that quantum information isn't the same information as the classical information. Thus quantum information is new kind of information. This fact was demonstrated by using two different communication systems. The first was a pure quantum channel and the second one was divided into three parts: quantum channel, classical and quantum. There were two types of devices, the measuring and preparing. Both were described by using operators. There were shown that these channels are identical when the preparing device exists. To realize it, there is a need to clone the original state but it is impossible. The next possibility is in receiving a large of number particles which are in the same quantum state.

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