# SPECTRAL ANALYSIS OF RABBIT ELECTROGRAM

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## ABSTRACT

Our experiments are oriented in detection the T-wave alternans of rabbit electrogram with synthetically evoked the cardiac diseases. In each system for ECG processing usually implemented block of R-peak detection. Most of them are based on filtering. The T-wave suppression and the R-wave amplification is results of the filtration. This paper will deal about the differences between spectral properties of QRS complexes and T-wave of rabbit electrogram.

## 1 PARAMETRIC POWER SPECTRUM ESTIMATION METHOD

For spectral analysis of non-stationary signals were used parametric method. It gives a better results in frequency and time resolution than classical nonparametric spectrum estimation method. Nonparametric method can be simple computed via DFT. However, these methods require the availability of long data records to achieve the sufficient frequency resolution. On the other hand short time segments we need to analyze. Parametric methods for power spectrum estimation is useful method for analyzing short non-periodic sequences.

The parametric methods are based on modelling the data sequence $x(n)$ as the output of a linear system characterized by a rational system function form [1]

$$H(z) = \frac{B(z)}{A(z)} = \frac{\sum_{k=0}^{q} b_k z^{-}k}{1 + \sum_{k=1}^{p} a_k z^{-k}}. \tag{1}$$

The corresponding difference equation is

$$x(n) = -\sum_{k=1}^{p} a_k x(n-k) + \sum_{k=0}^{q} b_k w(n-k), \tag{2}$$

where $w(n)$ is the input sequence and $x(n)$ presents the output data. If $w(n)$ is characterized as a stationary random process than the output sequence is also assumed as a random process. In such a case the power density spectrum is

$$\Gamma_{xx}(f) = |H(f)|^2 \Gamma_{ww}(f), \tag{3}$$

where $\Gamma_{ww}(f)$ is the power density spectrum of input sequence and $|H(f)|$ is the frequency response of the model (1). In case of input sequence is zero-mean white noise with auto-correlation $\gamma_{ww}(m) = \sigma_w^2 \delta(m)$, the equation of power density spectrum (3) is simply

$$\Gamma_{xx}(f) = \sigma_w^2 |H(f)|^2 = \sigma_w^2 \frac{|B(f)|^2}{|A(f)|^2}, \tag{4}$$

where $\sigma_w^2$ is the variance (e.g., $\sigma_w^2 = \sum\{|w(n)|^2\}$). The spectrum estimation consist of two steps. For first, it has to be estimated parameters $a_k$ and $b_k$ of the model for given data $x(n)$. Then we can compute the power density spectrum according to (4).

The model described in (1) is called an autoregressive-moving average (ARMA) process of order $(p,q)$. If we set the $q = 0$ and $b_0 = 1$ we get simplified function $H(z) = 1/A(z)$ called autoregressive (AR) models of order $p$.

For these experiments were used AR model and Burg method for AR model parameters determination. The Burg method use a forward and backward linear prediction, based on a lattice structure of FIR filter.

## 2 FORWARD AND BACKWARD LINEAR PREDICTION

The linear prediction has many practical applications. The Burg method uses forward and backward linear prediction for AR model parameters determination. By other words, the linear prediction is predicting the value of a stationary random process in forward and backward time. This leads to lattice structure FIR filter, see fig. (1). The parameter $K$ is

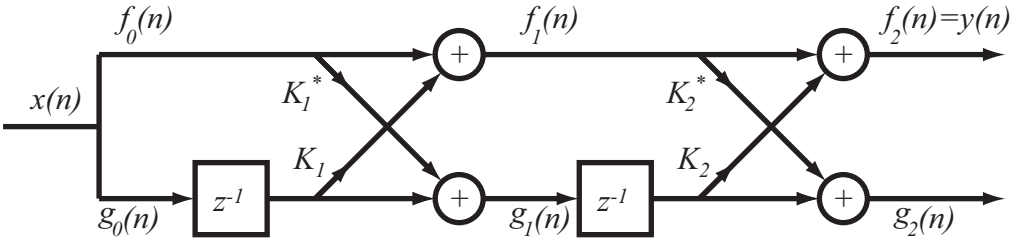


Figure 1: Two-stage lattice filter

called reflection coefficient. For $m$-stage lattice filter can be written recursive equation:

$$\begin{aligned} f_0(n) &= g_0(n) = x(n) \\ f_m(n) &= f_{m-1}(n) + K_m g_{m-1}(n-1), \quad m = 1,2,\ldots,p \\ g_m(n) &= K_m^* f_{m-1}(n) + g_{m-1}(n-1), \quad m = 1,2,\ldots,p. \end{aligned} \tag{5}$$

Function $f_m(n)$ is the forward and the $g_m(n)$ is backward prediction error in $m$-stage lattice structure.

**Forward linear prediction** is process to get estimation value $\hat{x}(n)$ of future value $x(n)$ with knowledge of previous $p$ values $x(n-1)$, $x(n-2), \ldots,$ $x(n-p)$. This can be presented as linear combination (6) and the prediction error $f_p(n)$ as the difference between the real value $x(n)$ and estimation of $\hat{x}(n)$ as (7):

$$\hat{x}(n) = -\sum_{k=1}^{p} a_p(k)x(n-k), \tag{6}$$

$$f_p(n) = x(n) - \hat{x}(n) = x(n) + \sum_{k=1}^{p} a_p(k)x(n-k), \tag{7}$$

where $a_p(k)$ are weights in the linear combination. It can be seen that the equation (7) is similar to FIR linear filter with input sequence $x(n)$, output sequence $f_p(n)$ and parameters of the filter $a_p(0), \ldots, a_p(p)$, (the parameter $a_p(0) = 1$). This filter is called a prediction error filter. Transmission function of this filter is:

$$A_p(z) = \frac{F_p(z)}{X(z)} = \sum_{k=0}^{p} a_p(k) \cdot z^{-k}. \tag{8}$$

**Backward linear prediction** is reversely defined problem. We have data from stationary random process $x(n), x(n-1), \ldots, x(n-p+1)$ and we have to predict the value $x(n-p)$. The backward linear prediction equation of order $p$ and the backward linear error $g_p(n)$ may be determined similarly as in previous paragraph:

$$\hat{x}(n-p) = -\sum_{k=0}^{p-1} b_p(k)x(n-k), \tag{9}$$

$$g_p(n) = x(n-p) - \hat{x}(n-p) = x(n-p) + \sum_{k=0}^{p-1} b_p(k)x(n-k), \tag{10}$$

where $b_p(k)$ is weighting coefficients of backward linear prediction. This prediction may be realized similarly by FIR filter as a lattice structure. If we compare equations (7) and (10), we deduce that weights $b_p(k)$ are in reverse numerical order and complex conjugates of $a_p(k)$ is necessary for complex valued data:

$$b_p(k) = a_p^*(p-k), \quad k = 0, 1, \ldots, p. \tag{11}$$

Transmission function of the backward linear system and from (11) we get an equation:

$$B_p(z) = \frac{G_p(z)}{X(z)} = \sum_{k=0}^{p} b_p(k) \cdot z^{-k} = z^{-p} A_p^*(z^{-1}). \tag{12}$$

Transformation the equation (5) into $z$ domain and substituting (8) and (12) may be derived following equation [1]:

$$\begin{bmatrix} A_m(z) \\ B_m(z) \end{bmatrix} = \begin{bmatrix} 1 & K_m \cdot z^{-1} \\ K_m^* & z^{-1} \end{bmatrix} \cdot \begin{bmatrix} A_{m-1}(z) \\ B_{m-1}(z) \end{bmatrix}. \tag{13}$$

The Burg method is worked on minimizing the forward and backward prediction error. It uses the lattice structure of FIR filter and compute the reflection coefficients $K_m$, $m = 1, 2, \ldots, p$. From it we can compute the $a_p(k)$ weighting parameters.

## 3 BURG METHOD FOR AR MODEL PARAMETERS DETERMINATION

The method devised by Burg, it is based on minimization of the forward and backward errors in linear predictors. The reflection coefficients may be obtained by the following equation:

$$\hat{K}_m = \frac{-\sum_{n=m}^{N-1} f_{m-1}(n) g^*_{m-1}(n-1)}{\frac{1}{2}\sum_{n=m}^{N-1}\left[|f_{m-1}(n)|^2 + |g_{m-1}(n-1)|^2\right]}, \quad m = 1,2,\ldots,p. \tag{14}$$

The $a_m(k)$ parameters may be compute by Levinson-Durbin recursion derived from (13):

$$a_m(k) = a_{m-1}(k) + K_m a^*_{m-1}(m-k), \quad k = 1,\ldots,m-1, \quad m = 1,\ldots,p. \tag{15}$$

Finally the power spectral density may be computed as a fraction:

$$P^{BU}_{xx}(f) = \frac{\hat{E}_p}{\left|1 + \sum_{k=1}^{p} \hat{a}_p(k) e^{-j2\pi fk}\right|^2}, \tag{16}$$

where prediction error may be expressed recursively as $E_m = (1-|K_m|^2)E_{m-1}$. The value $E_0$ is the variance of the input signal.

## 4 IMPLEMENTATION

The Burg method were used for spectral analysis of two types of ECG signals. First were obtained from CSE bank of common standards in ECG signals and the second, rabbit electrogram, were obtained from the experiments on isolated heard by electro-optical way [2], sampling frequency of bow signals were $f_s = 500\,\text{Hz}$. This paper has been writ-

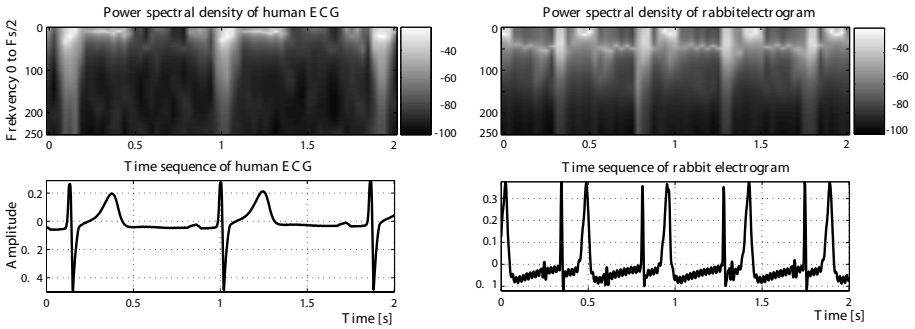


Figure 2: Power spectral density. AR parameters $p = 4$, $N = 30$.

ten with connection to T-wave alternans (TWA) and QRS detector. All known studies of TWA and QRS detectors uses spectral properties of human ECG. Filtering by band-pass filter13 to 24 Hz can be marked off the QRS complex. However, the rabbit electrogram has another spectral density, which hinder correct detection.

On fig. 2 is shown evolution of power spectral density. In case of rabbit electrogram is more difficult to isolate the QRS complex by FIR filtering, because the power spectral density of R wave and T wave is similar especially in higher frequency, as can be seen on fig. 3. This figure give evidence about power spectral density of R and T wave of

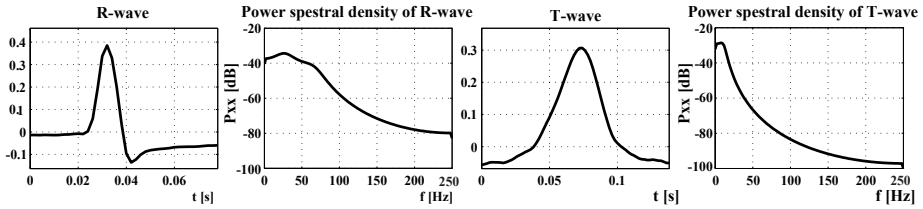


Figure 3: Power spectral density of R-wave (left) and T-wave (right), $p = 4$.

rabbit electrogram. For first it fulfil a premise that T-wave is low frequency instead of high frequency R-wave. With increasing frequency the power spectral density is slightly falling. From frequency approximately $f = 100$ to 250 Hz is probability of appearance each frequency component of T-wave about 10 times lower than R-wave, however the energy of R-wave in this frequency range is too low, in most of cases under press of noise, and the detection is also difficult.

## 5 CONCLUSION

The power spectral density of the rabbit electrogram give information about appearance of each frequency components. The filtering by high-pass or any other type of filter for detection of R-peak in electrogram isn't sufficient. The reason of this is the problem described in previous paragraph. For the best detection is better to use a decomposition by the filters bank, e.g. wavelet transform. The detection may start on the lowest frequency band, where the energy on R and T waves is the biggest. In higher bands the energy falling down, but energy of T-wave is dropping more quickly. The algorithm for R-peak detection is a theme of future research.

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