TEST OF OPTIMUM AND SOLUTION IMPROVEMENT

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ABSTRACT

The submitted article describes the optimization algorithms for decrease of expanses for transportation problem. After the definition of transportation problem and after the downloading of some methods of its solution I indicate a special method for optimization which does not need the knowledge of simplex method.

1 INTRODUCTION

The transportation problem is one of problems of linear programming but very often it solved by other special methods as for example by NW-corner method or VAM-method. The reason of this consists in a very big number of zeros in the system constraining conditions. After the first step of these methods we do not receive the optimal value of the objective function. It is necessary to do a betterment of solution. Such a method is given in this paper.

In the second phase of solving of transportation problem we ask how to recognize that our solution is optimal i.e. minimal optionally that the solution can be improved and how to receive the better solution. We give an algorithm for the conciliation of a better solution.

2 ROW AND COLUMN NUMBERS AND THE TEST OF OPTIMUM

We suppose non-degenerated transportation problem. At the solution of non-degenerated transportation problem with m suppliers and n customers $(m+n-1)$ cells are occupied. Simultaneously at least one stone (occupied cell) is lying in every row and every column.

We introduce row numbers u_i, $i = 1, 2, \ldots, m$ and column numbers v_j, $j = 1, 2, \ldots, n$ satisfying the following equations for every stone (occupied cell):

$$u_i + v_j = c_{ij} , \qquad (1)$$

where c_{ij} are the original costs of a given transportation problem. Let us solve this system. We stand ahead of a problem. The number of unknowns i.e. the number of row and column numbers is $m + n$ and we have only $m + n - 1$ equations. So, there exists one degree of freedom and that is not a problem but on contrary a preference. We can choose an arbitrary row and column numbers and assign to it an arbitrary value, preferably zero. We do this choice in that row or column which contains maximal number of stones. After the calculus of all the row and column numbers with the aid of (1) we work with waters (non occupied cells). We count new parameters c'_{ij} for waters using the following system of equations:

$$u_i + v_j = c'_{ij}\,. \qquad (2)$$

We take down these parameters into waters as that we write them into the left bottom corner. The system of identities $c_{ij} = c'_{ij}$ holds for stones. Therefore we do not write the new parameters into the cells which are stones. We exemplify computational procedure and assigning of values of row and column numbers of our example. We come out from the Tab. 2 obtained using the North – West corner method. We enlarge the table by addition of one row and one column. Into the column and row headings there we write variables v_j and u_i and into cells of columns and rows, concrete values of these numbers. We eke out the waters with differences $c'_{ij} - c_{ij}$. These differences are written into the left upper corner of waters. When all these differences are negative or equal to zero then we have the optimal solution. When at least one of these differences is positive there exists a water P_{ij} for which $c'_{ij} - c_{ij} > 0$ we have not optimal solution yet. In the case of greater number of positive differences we choose the water (cell) with maximal value of difference. We see that in our example the water with this property is P_{21}. Now the process of optimizing (minimizing) is starting.

Suppliers	Customers				Capacity	u_i
	K_j					
S_1	20 **250**	14 **60**	6 11 17	2 12 14	**310**	-1
S_2	15 6 21	15 **40**	18 **150**	15 **10**	**200**	0
S_3	12 17 29	11 12 23	7 19 26	23 **190**	**190**	8
Demands	250	100	150	200	700	
v_j	21	15	18	15		

Tab. 1: *Optimization process – initial state*

3 PATTERNS OF CHANGE METHOD

The process of optimizing starts as that we mark the cell with the greatest difference $c'_{ij} - c_{ij}$ by the sign + and we do the following table search: We look for a polygon created by apexes which are represented by stones and the apex assigned by the sign +. Simultaneously its sides are required to be horizontal or vertical only. Anyway we start in the cell with sign + and continue downstream the row or the column so on down to stone which will be the new apex of our polygon. This stone must satisfy the following condition. When we start from it, now in vertical direction (when we came to it in the horizontal way) we must find an other stone from which there exists a path to the next stone now in horizontal sense and so on till we come back into our starting point with the sign +. Simultaneously the starting cell is assigned by +, the next cell (stone) which is simultaneously the apex of the polygon will get the sign -. The next stone (apex) gets the sign + and so one. After going through the polygon we get back to apex which changes its signs. We construct the polygon for our example.

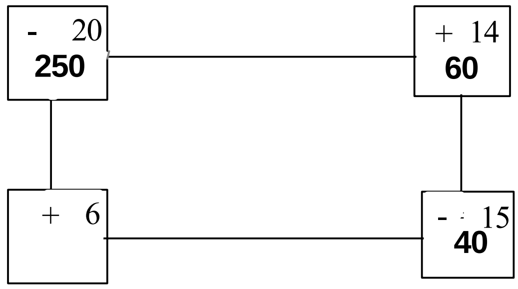

Fig. 1: *Polygon for the situation on tab. 1*

The polygon for the situation on tab. 1 is given in the figure 1. The picture idealizes the concrete situation in the fact that the sides are of the zero length in our polygon. The apexes of our polygon are neighboring stones in the table in the concrete. We return to the sequence of signs which is constructed as follows: As we know first apex which has sometimes the notation H has the sign +. The second one has the sign -. The third one has the sign + and so on. After some steps we come back to the apex H. Very important part is played by the subset of that apex which are denoted by sign -. We study this subset, more better said the subset of date content of this stones. The set of values in negative apexes is:

$$\{ 40, 250\}$$

The minimal value in this set is 40. We chose this smallest value and we add it to the content of apexes (stones) which were denoted by sign + and we subtract this smallest value from the content in apexes (stones) with sign -. The result of optimizing rearrangements is given at the Fig. 2.

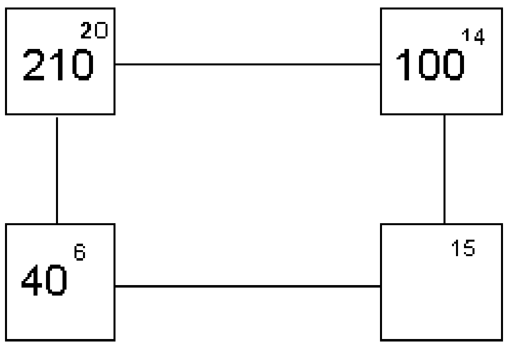

Fig. 2: *The result of optimizing rearrangements*

We obtain after the first optimizing the following tab. 2

Suppliers	Costumers				Capacity
	K_1	K_2	K_3	K_4	
S_1	20 **210**	14 **100**	11	12	**310**
S_2	6 **40**	15	18 **150**	15 **10**	**200**
S_3	17	12	19	23 **190**	**190**
Demands	250	100	150	200	700

Tab. 2: *Optimization process – step 1*

We must enumerate if it is the optimal solution. For this reason we extend the table rather one number row and one number column.

Suppliers	Customers				Capacity	u_i
	K_1	K_2	K_3	K_4		
S_1	20 **210**	14 **100**	21 11 32	17 12 29	**310**	14
S_2	6 **40**	-15 15 0	18 **150**	5 **10**	**200**	0
S_3	-3 17 14	-4 12 8	7 19 26	23 **190**	**190**	8
Demands	250	100	150	200		
v_j	6	0	18	15		

Tab. 3: *Optimization process – step 2*

We see that the process of optimizing is not finished and that the water cell with maximal difference is the cell P_{13}. Now the apexes of the according polygon are the cells: P_{11}, P_{21}, P_{23}. The set of values in negative apexes is:

$$\{ 210, 150 \}.$$

The minimal value in this optimizing step is 150. Hence we add the value 150 into apexes with sign + and we subtract this value in apexes with the sign -. We obtain:

Suppliers	Customers				Capacity	u_i
	K_1	K_2	K_3	K_4		
S_1	20 **60**	14 **100**	11 **150**	17 12 29	**310**	0
S_2	6 **190**	-15 15 0	18	15 **10**	**200**	-14
S_3	-3 17 14	-4 12 8	-14 19 5	23 **190**	**190**	-6
Demands	250	100	150	200		
v_j	20	14	11	29		

Tab. 4: *Optimization process –step 3*

We see that we did not obtain the optimal (minimal) solution. We calculated new row and column numbers and also the new differences and we see that maximal value of the differences is in the water cell P_{14} is 17 and the polygon contains apexes (stones) P_{11}, P_{21}, P_{24} or of apexes P_{13}, P_{23}, P_{24} and the initial water cell. For both the polygons the value which will be recounting is equal to 10.

After a finite number of optimizing steps we receive the stage described in the following tab. 5.

Suppliers	Customers				Capacity	u_i
	K_j					
S_1	-11 20 9	-10 14	11 **110**	12 **200**	**310**	-8
S_2	6 **200**	-14 15 1	-10 18 8	-6 15 9	**200**	-11
S_3	17 **50**	12 **100**	19 **40**	-3 23 20	**190**	0
Demands	250	100	150	200	700	
v_j	17	12	19	20		

Tab. 5: *Optimization process – step 4*

We see that all the differences $c'_{ij} - c_{ij}$ calculated for water cells with the aid of the last row and column numbers are negative. It would be sufficient for optimality to receive non positive differences (we admit also differences equal to zero).

4 CONCLUSION

The given method allows to do the optimizing of the prices for transportation problem without the usage of simplex method. It can be done for small systems using a hand calculator. The greater and great systems require to do construct a program for calculation on computer.

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