PIEZOELECTRIC SENSORS: BASIC MODELS

Ing. Petr SEDLÁK, Doctoral Degree Programme (1)
Department of Physics, FEEC, BUT
E-mail: xsedla49@stud.feec.vutbr.cz

Ing. Štěpán HEFNER, Doctoral Degreee Programme (1)
Dept. of Physics, FEEC, BUT
E-mail: xhefne00@stud.feec.vutbr.cz

Supervised by Prof. Josef Šikula

ABSTRACT

Non-destructive testing is a phenomenon of this time and acoustic emission (AE) is a one of NDT methods. Growing interest in AE method also increase sensor requirements. Sensor models and their simulations assist in fulfilling requirements. The paper presents sensors modelling, strictly speaking modelling of piezoceramics, which are most widely used in design of AE sensor. The basic models and their properties are mentioned.

1 INTRODUCTION

Since piezoelectricity discovery in 1880, piezoelectric materials have found their way into many scientific and commercial applications. By definition, a piezoelectric material generates a charge when put under pressure, and will show a change in volume when an electrical field is applied.

Fig. 1: *piezoelectric sensor*

So piezoceramic can be used as a transducer material for transforming electrical energy into mechanical energy and vice versa. If a mechanical vibration is applied, then a charge of proportional size and same frequency will be generated. It is one of many applications utilize the high frequency characteristics of piezoceramic materials, the piezoelectric sensors principle.

Piezoelectric sensor modeling is useful not only for evaluation of cracks detection and other sensing, but also it is useful for recognition of sensors parameters. This knowledge is very important to design of future sensors.

2 BASIC MODELS

The parameters of piezoelectric sensors are closely related to properties of materials from which they are made. The major material characteristics are elasticity, permitivity and piezoelectricity. Also it is important to know natural frequencies. Three different mathematical models are mentioned. We suppose that these models relation can describe complete view of sensor.

2.1 MECHANICAL MODEL

For simplicity, piezoceramic disc is considered as model to study. In first time, task can be as disc vibration [1]. The resonance frequencies are computed and their influence on sensor electrical characteristics is researched.

Equation of motion describes behaviour of a disc vibration. The differential equation in polar coordinates, {r, Θ}, governing the axial displacement of the plate, w, is

$$\nabla^4 w(r,\Theta) - \frac{\rho\, h\, \Omega^2}{D} w(r,\Theta) = 0\,, \tag{1}$$

which when written in extensor becomes:

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \Theta^2} \right)^2 w - \frac{\rho\, h\, \Omega^2}{D} w(r,\Theta) = 0\,, \tag{2}$$

where ρ is density of the plate material, w is plate displacement, Ω is natural frequency $D = \frac{E\,h^2}{3\,(1-\mu)}$ is flexural rigidity, E is Young’s modulus, μ is Poisson's ratio and 2h is the thickness of the plate.

After editing, a vibrating solution can be by equation:

$$w(r,\Theta) = \left[A_{m,n} J_m\left(\lambda_{m,n} \frac{r}{R} \right) + B_{m,n} J_m\left(i\lambda_{m,n} \frac{r}{R} \right) \right] \sin\left(m\Theta + \varphi_{m,n} \right), \tag{3}$$

where

$$\lambda = R\sqrt{\frac{\rho\, h\, \Omega^2}{D}}\,, \tag{4}$$

and where J_m is Bessel function first type.

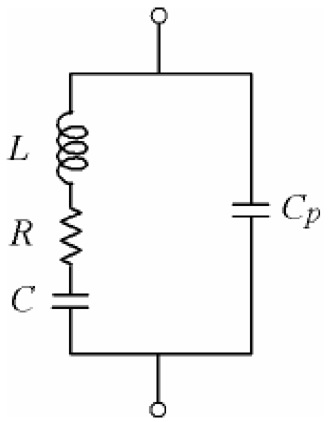
2.2 EQUIVALENT CIRCUIT (ELECTRICAL MODEL)

The base of piezoelectric sensors is usually piezoceramic segment. The equivalent circuit can describe its behavior (see Fig.2) [2]. The following two conditions are required for the circuit to accurately simulation piezoelectric resonance behavior:

- the value of C must be much smaller than Cp,
- C and Cp in parallel must equal the piezo's low-frequency capacitance.

The frequencies of the electrical serial resonance and parallel resonance are following equations (which assume a small series resistance R):

$$f_{rs} = \frac{1}{2\pi\sqrt{LC}}, \qquad f_{rp} = \frac{1}{2\pi}\sqrt{\frac{C_p + C}{LC_pC}}. \tag{5}$$



C_p vacuum capacitance

R the resistance caused by mechanical losses

C the capacitance of mechanical circuit

L the inductance of mechanical circuit

Fig. 2: *Equivalent circuit scheme of piezoelectric transducer*

Of course this scheme cannot be used as piezoelectric sensor equivalent circuit diagram (model). Electrical characteristics of piezoelectric sensor are based on impedance and phase measurement [3]. It is very important to known that piezoelectric sensor equivalent circuit is not possible to model by series or parallel combination of one RC circuit only (see Fig.3).

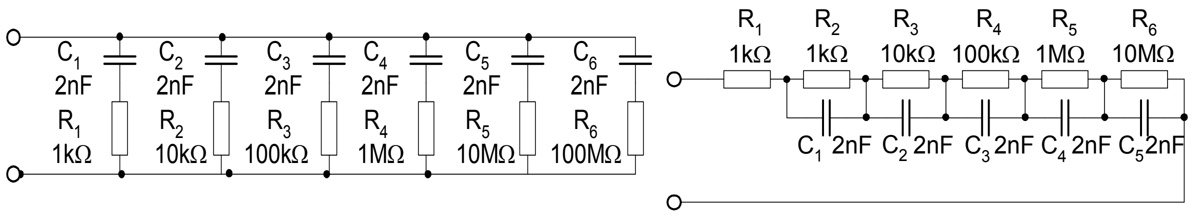


Fig. 3: *Sensor equivalent circuit, model 1 and model 2*

First equivalent circuit is based on parallel combination of series RC circuits and the second one is given by series combination of parallel RC circuits. Both models must be optimised for total capacitance and phase frequency dependence.

Equivalent circuit (electrical model) describe sensor electrical properties such as the admittance impedance characteristic and also signal to noise ratio.

2.3 PHYSICAL MODEL

There are two differential equations governing the behaviour of a piezoelectric continuum [4] - Newton's laws of motion (6) and the quasistatic approximation Maxwell's equation (7). Let us denote the volume of the resonator as the volume V . The time range, in which we solve the problem, is (0; τ).

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \frac{\partial T_{ij}}{\partial x_j}, \quad i = 1,2,3 \quad x \in V \quad t \in (0, \tau),. \tag{6}$$

$$\nabla D = \frac{\partial D_j}{\partial x_j} = 0\,, \tag{7}$$

where T is the stress tensor, D is the electric flux density vector.

The above equations are coupled by the piezoelectric equations:

$$\begin{aligned} \{T\} &= [c]\{S\} - [e]\{E\}, \\ \{D\} &= [e]\{S\} + [\varepsilon]\{E\}, \end{aligned} \tag{8}$$

where S is strain tensor, E is electric filed vector, c, e and ε are the stiffness, piezoelectric and permitivity tensors of quartz (these tensors are typical for each material).

3 FEM MODEL

FEM modelling is a one of possible mathematical relation between basic models. The fundamental assumption of finite element analysis [5] is that any continuous function, such as stress, strain or electric field, can be approximated by discretisation. The original volume is divided into elements, and within each element the function is constant or a simple function of position, either linear or quadratic. At each vertex of an element is a “node“ and the number of variables acting at each nodal site is called the “degree of freedom“ (DOF).

For piezoelectric there are four DOF at each node; UX, UY and UZ (3D displacement) and voltage. To each DOF there is a reaction force FX, FY, FZ to the displacement and charge Q to the voltage.

The electromechanical constitutive equations for linear behaviour of the elements are above (8). Application of the variational principle of finite element discretisation to the coupled finite element discretisation yields the following equation:

$$[M_{uu}]\ddot{u} + [C_{uu}]\dot{u} + \begin{bmatrix} K_{uu} & K_{u\phi} \\ K_{u\phi} & K_{\phi\phi} \end{bmatrix} .u = \begin{bmatrix} F \\ Q \end{bmatrix}, \tag{9}$$

where [K_{uu}] is mechanical stiffness matrix derived from [c] matrix, [$K_{u\phi}$] is piezoelectric stiffness matrix derived from [e] matrix, [$K_{\phi\phi}$] is dielectric stiffness matrix derived from [ε] matrix, [C_{uu}] is mechanical loss matrix, [M_{uu}] is inertia matrix derived from density and volume, u is displacement vector, ϕ is voltage vector, F is mechanical force vector, Q is charge vector.

4 CONCLUSION AND FUTURE WORK

There are four possible description of piezoelectric sensor mentioned in this paper. On the base of simply model and measuring, we suppose, that new type of model could be made. The experimental results (see Fig.4 and Fig.5) show that the sensitivity of the sensors is not only dependent on the sensors inherent features such as piezoelectric properties and geometry, but also on the admittance and impedance of the sensor (equivalent circuit) and naturally on attached electrical circuit. No mathematical relation between mechanical vibration and electrical characteristics was described till now. This description is necessary for piezoelectric model connection to an electrical circuit and for modelling signal to noise ratio. Our future work is to describe this relation.

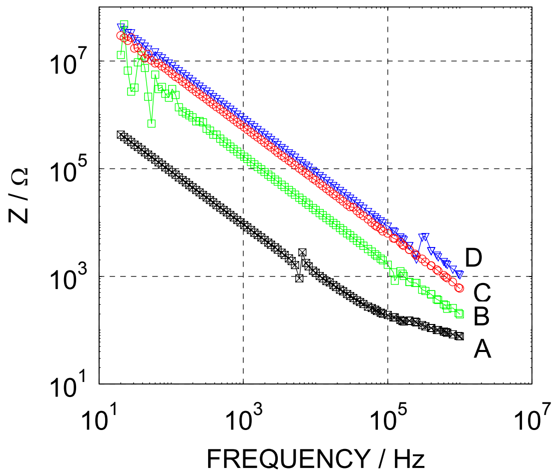


Fig. 4: *Frequency dependence of impedance four sensors*

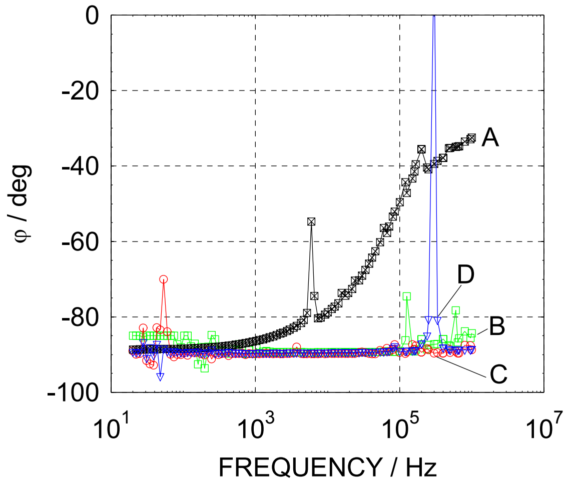


Fig. 5: *Frequency dependence of phase four sensors*

ACKNOWLEDGEMENTS

The research has been partially supported by grant GAČR 205/03/0071 and grant MSM 262200022.

REFERENCES

[1] Slavík, J., Stejskal, J., Zeman, V.: Základy dynamiky strojů, ČVUT, 1997

[2] Waanders, J. W.: Piezo-electric Ceramics, NV Philips 1991

[3] Šikula, J., Hájek, K., Štrunc, M., Majzner, J., Koktavý, P., Pavelka, J.: Electrical Noise of Piezoceramic Sensors

[4] Maryška, J., Rálek, P., Novák, J.: Application of FEM in Modelling of the Resonance Characteristics of Piezoelectric Resonators, Proc. of Algoritmy 2002

[5] Lowrie, F., Cain, M., Stewart, M.: Finite Element Modelling of Electroceramics, NPL Report, 1999