MODELING OF SPECIAL ELECTROMAGNETIC PROBLEM

Ing. Eva KADLECOVÁ, Doctoral Degree Programme (3)
Dept. of Electrical Power Engineering, FEEC, BUT
E-mail: kadleca@feec.vutbr.cz

Supervised by: Dr. Pavel Fiala

ABSTRACT

In technical praxis we often encounter coupled problems. There are two methods of computation: direct or indirect method. One of the possibilities how to solve these problems is to use the analogy between different physical models. This article demonstrates using the similarity of physical models for the modeling of special electromagnetic problem.

1 PARTIAL DIFFERENTIAL EQUATIONS OF BOUNDARY PROBLEMS

Common boundary problems can be formulated with help of the following equations Laplace's equation

$$\Delta u = 0 \tag{1.1}$$

Poisson's equation

$$\Delta u = f \tag{1.2}$$

Helmholtz's equation

$$\Delta u = c.u \tag{1.3}$$

Common heat - conduction equation (diffusional)

$$\Delta u = c.\frac{\partial u}{\partial t} \tag{1.4}$$

Schrödinger's equation

$$\Delta u = c.\frac{\partial u}{\partial t} + bu \tag{1.5}$$

Wave equation

$$\Delta u = c.\frac{\partial^2 u}{\partial t^2} \tag{1.6}$$

Thermal equation

$$\Delta u = a.\frac{\partial^2 u}{\partial t^2} + b\frac{\partial u}{\partial t} + c \qquad (1.7)$$

Biharmonic equation

$$\Delta\Delta u = 0 \qquad (1.8)$$

Where Δ is Laplace's operator defined as

$$\Delta = \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \qquad (1.9)$$

u is every function with continuous partial second derivatives in area Ω, that fulfills equation (1.10). *a,b,c* are constants dependent of material properties.

$$\Delta u = f(x_1, x_2, ..., x_n) \qquad (1.10)$$

2 SIMILARITY OF PHYSICAL MODELS

Common types of boundary problems formulated by Poisson equation:

Electrostatic field

$$-div(\varepsilon \cdot grad\varphi_e) = \rho \qquad (2.1)$$

Current field

$$-div(\gamma \cdot grad\varphi_e) = q_e \qquad (2.2)$$

Thermal field

$$-div(k \cdot gradT) = q \qquad (2.3)$$

3D static magnetic field without current

$$-div(\mu \cdot grad\varphi_m) = 0 \qquad (2.4)$$

2D static magnetic field with current

$$-div(\nu \cdot gradA_{x,y}) = J_z \qquad (2.5)$$

Where ε is permittivity, ρ is charge density, φe is electrical potential, γ is conductivity, q_e is current source (in current field), *k* is heat-carrying capacity, *q* is heat generation rate per unit volume (in thermal field), *T* is temperature, μ is permeability, φ_m is scalar magnetic potential, ν is reluctivity, J_z is z component of current density, $A_{x,y}$ are x, y components of magnetic potential.

3 MODELING OF TECHNICAL PROBLEMS

Some technical problems cannot be solved as independent physical problems. In the technical praxis different types of fields are combined as described by equations (1.1) –(1.4). One of the existing problems is for example to solve Poisson's equation and use it to model simple light problem, see 3.11. (materials $\neq f(\lambda)$, $\lambda \in$ <0,1 μm – 100 μm >).

3.1 MODELING OF LIGHT SOURCE

A necessary part of the design process of light sources is the modeling and experimental verifying of results. Models based on radiation principle are counted amongst the most accurate mathematical models of light sources counts. One possibility is to use standard one–purpose programs; but another possibility offers us usage of sophisticated numerical methods, among them falls the finite element method, for example program ANSYS. In ANSYS is in area of thermal field analysis solved radiation.

It is possible to use standard program tools in the ANSYS program – modeling, discrimination to net of elements, solvers, evaluation and interpretation of results. The centre of the whole problem lies in the transformation of thermal field quantities into optical quantities. This can be done according to general rules described in book [2]. In the following text the basics of modeling the primitive light problem are described. The verification of the model of light source is done by experiment. Following these results it is possible to continue in modeling other problems such as hollow light guides and their applications.

3.2 MODEL BUILDING

The formulation of the basic thermal model is based on the first law of thermodynamics

$$q + \rho c \mathbf{v} \cdot divT - div(k\, gradT) = \rho c \left(\frac{\partial T}{\partial t} \right) \qquad (2.6)$$

where q is specific heat, ρ is specific weight, c is specific solidification heat, T is temperature, t is time, k is coefficient of calorific conduction, v is velocity of flow. This model can be with respect to application of Snell's principles simplified into form

$$q - div(k\, gradT) = \rho c \left(\frac{\partial T}{\partial t} \right) \qquad (2.7)$$

According to Stefan-Boltzmann principles, heat transfer by way of radiation between surfaces with relative indexes i, j is formulated as

$$q_{ri} = \sigma \varepsilon_i A_{i,j} S_i \left(T_i^4 - T_j^4 \right) \qquad (2.8)$$

where q_{ri} is specific heat transferring from surface with index i, σ is Stefan-Boltzmann's constant, ε_i is emissivity of surface, $A_{i,j}$ is projection factor of surface with index i to surface with index j, Si is area of surface with index i, T_i, T_j are temperature of surfaces i, j. Picture number 1 shows relation between quantities in equation (2.8). Projection factor $A_{i,j}$ is determined as

$$A_{i,j} = \frac{1}{S_i} \int_{S_i} \int_{S_j} \frac{\cos \vartheta_i \cos \vartheta_j}{\pi r^2} dS_j dS_i \qquad (2.9)$$

When the projection factor is determined, it is possible to use the Gallerkin's principles for converting this problem into the model (2). Marginal and initial conditionsmust be respected.

$$[K]\{T\} = \{Q\} \qquad (2.10)$$

where K is coefficients matrix, T is columnar matrix of searched temperatures, Q is columnar matrix of heat sources. From temperature T is determinate thermal flow T_f as

$$T_f = -(k\, gradT) \tag{2.11}$$

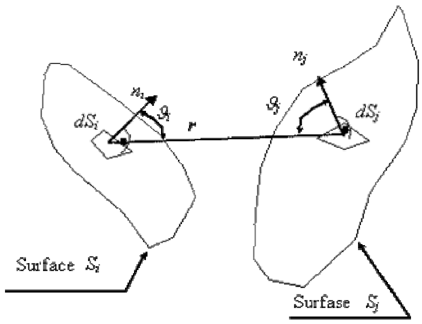

Fig. 1. Determination of projection factor Ai,j

For radiation principles are elements of column matrix of heat sources Q determined as

$$Q_{i,j} = S_i A_{i,j} \varepsilon_i\, \sigma\left(T_i^4 - T_j^4\right) \tag{2.12}$$

and after adjustment for mathematical model (2) is

$$Q_{i,j} = \overbrace{S_i A_{i,j} \varepsilon_i\, \sigma\left(T_i^2 - T_j^2\right)\left(T_i + T_j\right)}^{K}\left(T_i - T_j\right) \tag{2.13}$$

The heating model will be used for modeling of light problem using Snell's principles in optics. To light source with intensity of lightning *E (lx)* corresponds with equivalent heat quantity density of heat flow *q´´*, light flow *Φ(lm)* corresponds equivalent quantity - heat flow *q´*.

3.3 MODEL OF SIMPLE LIGHT SOURCE IN PROGRAM ANSYS

The geometrical model was built in the program by standard means. The mathematical model is created with help of the automated mesh generator creating elements and nodes. The used element is SOLID70.

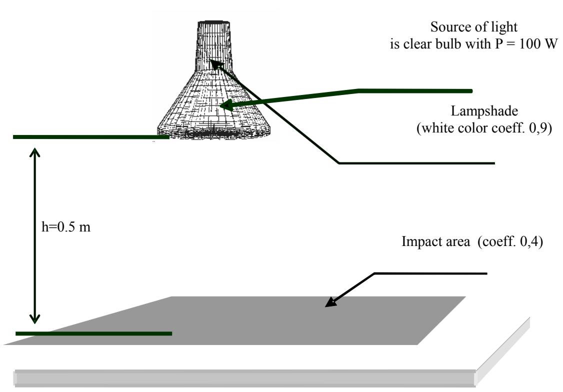

Fig. 2. Geometrical model of the problem

Fig. 2. describes characteristic geometrical shape of the model. It consists of a

lampshade source, the source is a classical bulb P = 100 W. The bulb is made of pure glass and wolfram fiber. We seek the dispersion of light flow in modeled area.

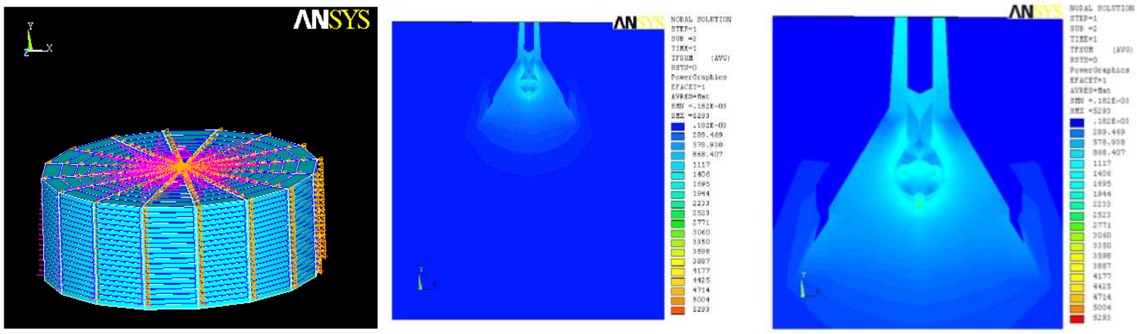

Fig. 3. Geometrical model of the problem and results of solution

3.4 RESULTS OF STATIC ANALYSIS OF THE MODEL

In **Fig.3.** and **Fig.4.** the values of light flux in the area of the modeled problem are shown. Marginal conditions are ideally dark walls at r = 1 m distance from the light. Results retrieved by the numeric modeling were verified by experimental measurement. Differences were between (5 – 20) % from results of the numerical modeling.

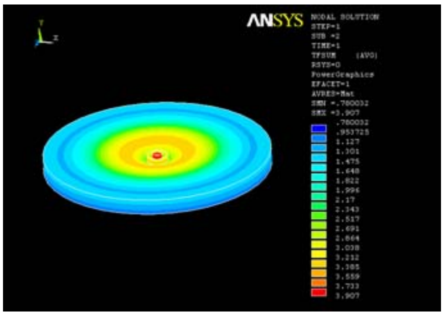

Fig. 4. Results on the surface under the modeling light

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