THE OPTIMALIZATION OF ELECTROMAGNET'S CORE

Ing. Tomáš KERLIN, Doctoral Degree Programme (1)
Dept. of Electrical Power Engineering, FEEC, BUT
E-mail: kerlin@kn.vutbr.cz

Supervised by: Dr. Miroslav Patočka

ABSTRACT

This paper focuses on the dimensional optimalization of the levitation electromagnet's core, which has U shape. We optimize the core to minimal volume of the core+coils and to minimal leakage flux. Then we suggest the way to merge these two criterions together. The results of an ANSYS FEM analysis are also proposed.

1 INTRODUCTION

One of the main objective of my disertation work is to design and create a levitation electromagnet. The first step was to derive a mathematical model of electromagnet which is proposed in e.g. [1]. The second step is an attempt to find out the best dimensions of the core regarding to input variables, which are the requested attractive force and the air gap. The whole electromechanical system consists of the DC electromagnet, a 2 quadrant current convertor, a state regulation and a measurement of the current and air gap to feedback control. The arrangement of the electromagnets with marked dimensions you can see on the fig.1.

There are Joule, leakage, hysteresis and eddy-current losses in the core. Thanks to one-way flow of magnetic flux are the hysteresis losses negligible. The core is composed from insulated metal sheets, thus the eddy–current losses are minimal too. The minimal volume of the coil is required, because the volume is proportional to losses [1], hence the smaller volume the smaller losses in the cupper. Similarly it is with the magnetic flux, the more magnetic flux flow through both columns, the bigger attractive force.

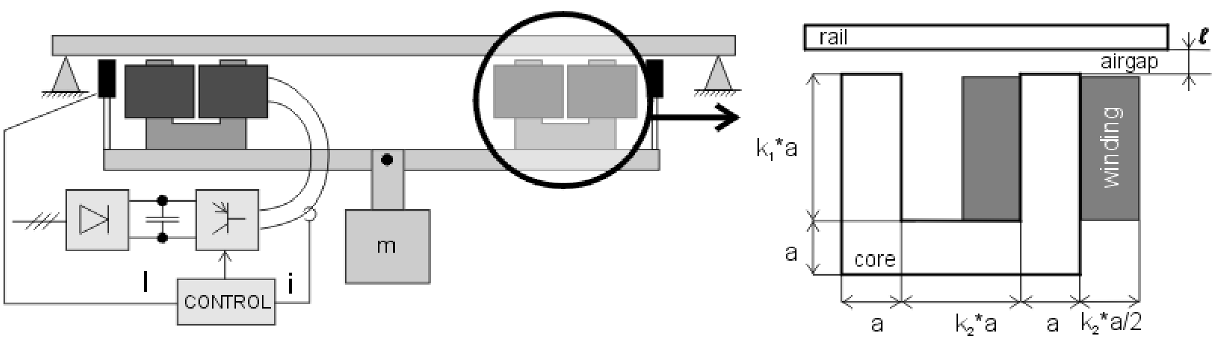

Fig. 1: *The arrangement of the system.*

2 THE OPTIMALIZATION TO THE MINIMAL VOLUME

At first we have to express the relation between the volume and core dimensions. It's usefull to work with the area of window S_0 and area of "iron" S_{Fe}, which good describe all important dimensions of electromagnet's core.

$$S_{Fe} = a^2 \qquad S_0 = k_1 a \cdot k_2 a \tag{1),(2}$$

From the fig.1 is obvious that the volume of the core V_{Fe} and volume of the winding V_{Cu} is defined:

$$V_{Fe} = 2 \cdot k_1 a \cdot a^2 + k_2 a \cdot a^2 + 2 \cdot a^3 = a^3 (2k_1 + k_2 + 2) \tag{3}$$

$$V_{Cu} = 2\left[(k_2 a + a)^2 \cdot k_1 a - k_1 a^3\right] = 2a^3 k_1 k_2{}^2 + 4a^3 k_1 k_2 \tag{4}$$

From the equations describing the mathematical model [1] we can isolate the relation for S_0 and from this notation is evident that the S_0 is completely defined by air gap l, which is constant in steady state and thereby S_0 is constant too.

$$S_0 = \frac{B}{\mu_0 \cdot \sigma \cdot k_{pl}} \left(2 \cdot l + \frac{l_{Fe}}{\mu_r} \right) = f_1\{\ l\} = konst. \tag{5}$$

The attractive force F is in steady state, thanks to constant current I and constant air gap l, constant as well and after solving for S_{Fe} we'll receive a relation defined as follows:

$$S_{Fe} = \frac{F \cdot \mu_0}{B^2} = f_2\{F\} = konst. \tag{6}$$

The sum of V_{Fe} and V_{Cu} is a function of dimensional variables k_1 and k_2. We would like to find the minimum of this function, but with the condition of constant S_0. The value of S_0 is set by necessary ampere-turns $N \cdot I$, which produce proper magnetic flux. The mentioned sum forms a function Ω:

$$\Omega(k_1; k_2; a) = V_{Fe} + V_{Cu} = 2a^3 k_1 + a^3 k_2 + 2 \cdot a^3 + 2a^3 k_1 k_2{}^2 + 4a^3 k_1 k_2 \tag{7}$$

The adjusted condition is: $k_1 k_2 a^2 - S_0 = 0$ (8)

The process of finding minimum of this function Ω is based on a geometric meaning of first derivation. The minimum is located at the point where this derivation is equel to zero. Because the minimum is tied with the condition of S_0, we should use some of the special methods to find the right minimum, for example a method of Lagrange's multiplikators. This method leads to the system of equations, whose results are stationary points. From these points we can choose the right minimum using a Sylvester's theorem. The result shows tab.1.

3 THE OPTIMALIZATION TO THE MINIMAL LEAKAGE FLUX

In this step we want to achieve the minimal leakage flux between core's columns. Considering fig.2 we can separate the total flux Φ into the useful and leakage flux, namely Φ_{US} and Φ_L respectively. The useful flux must flow from the first column through the airgap to the "rail" and again through airgap to the second column. Only this flux implies the force. Every flux flowing another way is leakage flux. We assume that a magnetic resistance of the core is negligable, because $\mu_{r,CORE}$ = 1000. The magnetic resistance of the airgap and colis we have to consider, because $\mu_{r,Cu}$ = $\mu_{r,AIR}$ = 1. The derivation of analytic formula consist in integration of leakage flux in axis x and y throught the whole "window" area.

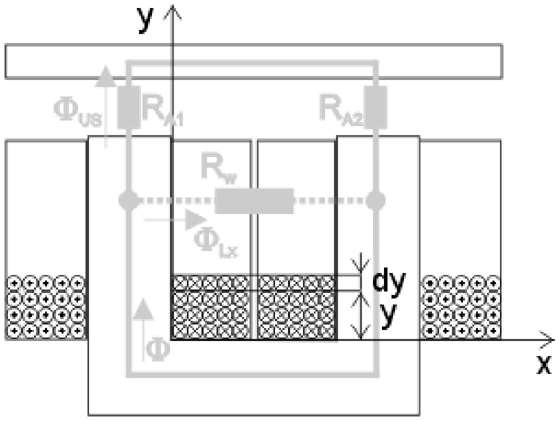

Fig. 2: *The calculation of leakage magnetic flux in X-axis.*

The overall magnetic voltage is defined $U_m = N \cdot I$ (9), then with regard to the fig.2 the unit voltage dU_m from element layer dy is:

$$dU_m = N \cdot I \frac{dy}{k_1 a} \tag{9}$$

The element increment of magnetic leakage flux in the direction of axis x is: $d\Phi_{Lx} = \lambda_m \cdot dU_m$ (10), where the magnetic conductivity λ_m of the volume in the "window" above the layer dy is:

$$\lambda_m = \mu \cdot \frac{S}{l} = \mu_0 \cdot \frac{a(k_1 a - y)}{k_2 a} \tag{11}$$

And after substitution (9), (11) to (10) and after integrating from 0 to $k_1 \cdot a$ we get:

$$d\Phi_{Lx} = \mu_0 \cdot \frac{a(k_1 a - y)}{k_2 a} \cdot N \cdot I \cdot \frac{dy}{k_1 a} \tag{12}$$

$$\Phi_{Lx} = \frac{\mu_0 \cdot N \cdot I}{k_1 k_2 a} \cdot \int_0^{k_1 a} (k_1 a - y) dy = \frac{1}{2} N I \mu_0 a \cdot \frac{k_2}{k_1} \tag{13}$$

It can be shown by analogy that if we neglect the dimension of the airgap l owning to the domension $k_1 \cdot a$, the leakage flux in the direction of axis y is:

$$\Phi_{Ly} = \frac{1}{2} N I \mu_0 a \cdot \frac{k_1}{k_2} \tag{13}$$

And then the total leakage flux is the sum of the fluxes in both axises.

$$\Phi_L = \Phi_{Lx} + \Phi_{Ly} = \frac{1}{2} N I \mu_0 a \cdot \left(\frac{k_1}{k_2} + \frac{k_2}{k_1} \right) \tag{14}$$

The minimal Φ_L comes on when $k_1 = k_2 = k_{F,MIN}$. So if the shape of the "window" is square, there will be minimal leakage flux. Again the results for are in the table 1.

4 MERGING BOTH MENTIONED OPTIMALIZATION TOGETHER

Both of the previous optimalizations are important for us. So we would like to have electromagnet optimized to both criterions together, thus we should tot them up. But if we want to do that, we have to standardize the functions Ω_Σ and Φ_L. The function Ω_Σ we will divide by concrete value of $\Omega_{\Sigma MIN}$, which we receive by establishing $k_{1V,MIN}$ and $k_{2V,MIN}$ into Ω_Σ. We get the function called $R_V(k_1;k_2)$ whose minimal value is 1 for $k_1 = k_{1V,MIN}$ and $k_2 = k_{2V,MIN}$. We have to do the same by analogy with Φ_L, the result function is $R_F(k_1;k_2)$.

Now we can tot up these functions. The coefficients C_V, C_F in equ.15 are real numbers between $<0;1>$ that say us how is the involved optimalization important for us. The bigger number, the more important for us. If the equation $C_V + C_F = 1$ will be kept, the theoretical minimal value of the function $R_\Sigma(k_1;k_2)$ will be 1, but this value can be achieved only if one of the coefficient is equal to zero. The bigger coefficient C_V over C_F , the closer to the optimalization of minimal volume we are.

$$R_\Sigma(k_1;k_2) = C_V \cdot R_V + C_F \cdot R_F \qquad (15)$$

$$R_\Sigma(k_1;k_2) = C_V \cdot \frac{1}{25}\left(2k_1 + k_2 + 2k_1k_2^2 + 4k_1k_2 + 2\right) + C_F \cdot \frac{1}{2}\left(\frac{k_1}{k_2} + \frac{k_2}{k_1}\right) \qquad (16)$$

As well as in chapter 3 we have to use the condition defined by equ. (8) and the same technique (the method of Lagrange‘s multiplikators) to find the minimum of the $R_\Sigma(k_1;k_2)$.

5 ANSYS FINITE ELEMENT ANALYSIS

Thanks to this analysis we are able to check correctness of our analytic calculation. We made analysis for various shapes of the core and observed a path of magnetic flux. The magnetic field was performed by 3D static nodal-based analysis. At first a SOLID5 element was used for the coils. The current excitation load was assigned as a current density of constant value. All surface-effects was neglected. The magnetic field was computed from vector magnetic potential using a SOLID97 element. We assume a linear material property of both “iron” parts, so the constant permeability was entered. A simulation part of surrounding air has a block shape and on his areas was placed the condition of zero vector of magnetic potential.

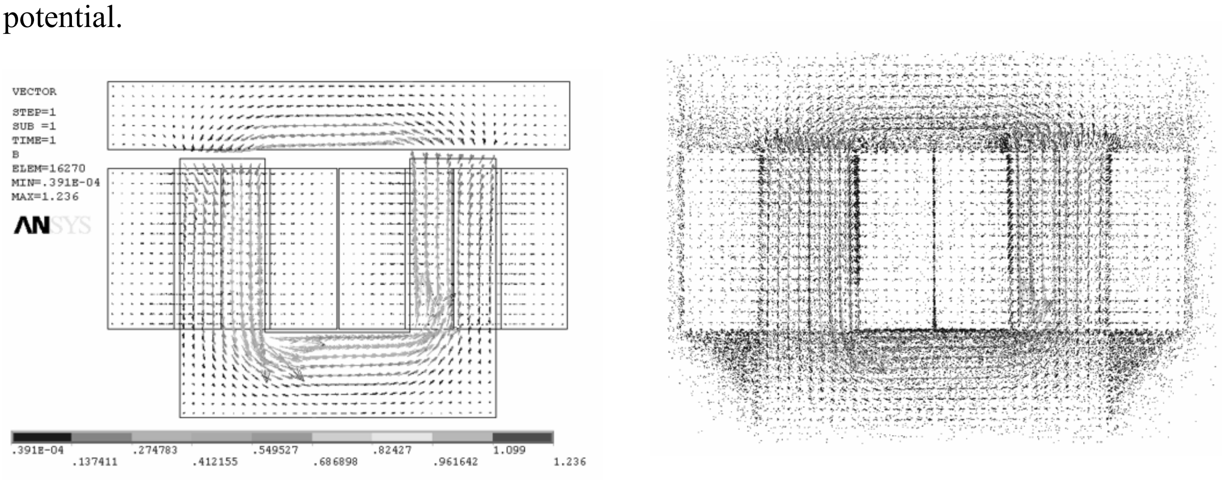

Fig. 3: *Some of the resultant pictures.*

6 CONCLUSION

In the following table are compared the results of all previous criterions. In spite of the fact that the dimension *a* is fully determined by requested force *F* (see equ. (1),(2)), we set this dimension *a = 5 cm* for our computation. Therefore *F* is determined by the dimension *a*. For the "joined" criterion we assume that both of the optimalizations are equally important for us, so $C_V = C_F = 0{,}5$.

	VOLUME OPT.	FLUX OPT.	VOL. + FLUX OPT.
k_1 [-]	3,00	1,66	1,78
k_2 [-]	1,92	1,66	1,55
ϕ_L/ϕ_{LMIN} [%]	178	100	101
$V_\Sigma/V_{\Sigma MIN}$ [%]	100	109	107
R_Σ [-]	1,932	1,044	1,039

Tab. 1: *The table of results.*

The volume opt. alone is useless, because then the core has great leakage flux (178% of the minimal possible leakage flux). The flux opt. alone is much better, it has of course minimal leakage flux and only 109 % of minimal volume. The final shape is very close to that one determined by flux optimalization, but the leakage flux is about 1 % bigger and the volume is about 2 % smaller.

The core's shape owning to first "volume" criterion is narrow and high, owning to minimal flux criterion is it exact square and owning to the joined one the core has compromise shape. This shape of the core is the best regarding our requiments.

Despite the ANSYS analysis was not perfect due to accepted simplifications, it confirms our previous results. The core with the dimensions received by joined criterion has the minimal leakage flux.

REFERENCES

[1] Gono, M.: Semestrální projekt 1, Brno, VUT FEKT ÚVEE

[2] Kerlin, T.: Projekt do předmětu DQV6, Brno, VUT FEKT ÚVEE