PRINCIPLE OF STEREOPHOTOGRAMMETRY - 3D POINT COORDINATES RECONSTRUCTION

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ABSTRACT

This paper deals with the principle of stereophotogrammetry, the principle of spatial point reconstruction from two 2D images. It describes an algorithm for 3D point reconstruction in case of free rotated cameras and different focal lengths.

1 INTRODUCTION

The traditional problem in computer vision is a 3D scene model reconstruction from a set of 2D images captured from different points of view. Having one 2D image and the centre of projection of its corresponding camera, we can construct a light-ray going through camera centre and the selected pixel in the image. On this light-ray, the spatial point, represented by the selected pixel in the image, lies. At least two images are required in order to reconstruct this 3D point; the light-rays intersect in the desired spatial position (see fig. 1). Having two pixels, each of them in a different image, which are projections of the same spatial point, this 3D point can be easily reconstructed using the camera light-rays intersection. Such two pixels create a *corresponding pair*, which is a basic element for the scene reconstruction process.

2 RECONSTRUCTION OF 3D POINT COORDINATES FROM TWO IMAGES

We consider two cameras, with parallel optical axes, and general point P in front of cameras. We establish Cartesian coordinate system with an origin in a principal point O_1 of left camera and axes x, y, z. Focal lengths of cameras is f_1 and f_2 and the spacing is b_x. The point P is projected into cameras planes as point p_1 and p_2. Parallax is defined as x-coordinates difference of P point projection into cameras plane

$$p_a = x_1 - x_2 \,, \qquad (1)$$

where

$$x_2 = x_2' \frac{f_1}{f_2} \,. \qquad (2)$$

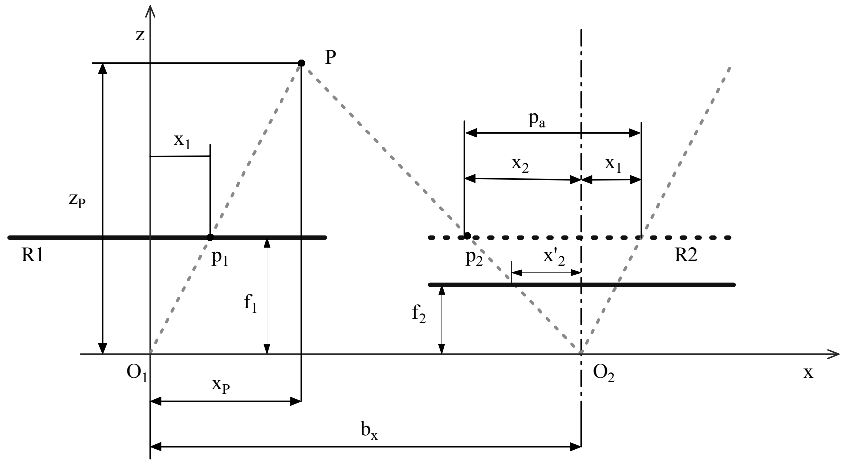

Fig. 1: *Principle of photogrammetry*

From a triangle similarity we obtain

$$\frac{x_1}{f_1} = \frac{x_p}{z_p}, \tag{3}$$

$$\frac{x_2}{f_1} = \frac{x_p - b_x}{z_p}. \tag{4}$$

From a subtraction of equations (3) a (4) we get

$$z_p = \frac{b_x \cdot f_1}{p_a}, \tag{5}$$

which is the basic equation of stereophotogrammetry.

For general point coordinates [*x,y,z*] we can write

$$x = \frac{b_x \cdot x_1}{p_a}, \tag{6}$$

$$y = \frac{b_x \cdot y_1}{p_a}, \tag{7}$$

$$z = \frac{b_x \cdot f_1}{p_a}. \tag{8}$$

where b_x is a camera base (spacing of cameras), f_1 is a camera focal length, p_a is a parallax.

These equations we can apply only in that case, when cameras are not rotated against each others and they are only translated in axis *x* (eventually in axis *y*). For generalized camera rotation is necessary to know or to determine intrinsic and extrinsic parameters of cameras, it is a coordinate of point O_2 [b_x,b_y,b_z] against an coordinate system origin and rotation angles φ, ω, κ of right camera against the left one (see fig.2). Then it is possible to transform this system back to an elementary case by the help of backward translation and rotation calculating.

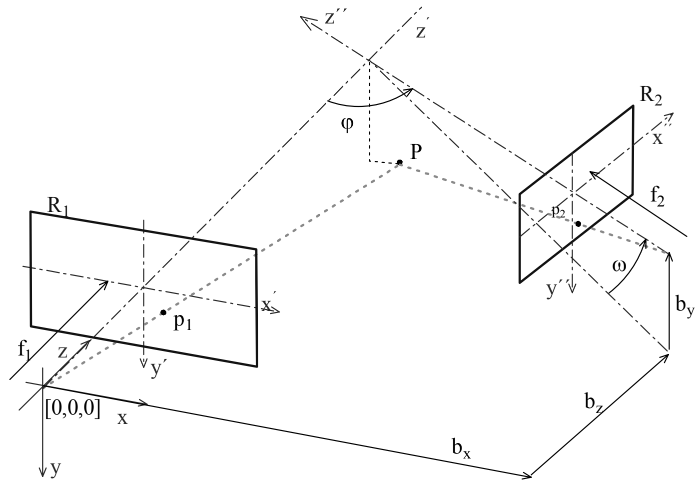

Fig. 2: *Situation of free camera rotation and translation*

2.1 TRANSLATION IN BOTH AXES *X* AND *Y*

In this case we could use equation (9), which takes into account a camera moving in both axes and converts this problem to basic condition (5).

$$z_p = f_1 \sqrt{\frac{b_x^2 + b_y^2}{p_x^2 + p_y^2}} \tag{9}$$

2.2 TRANSLATION IN AXIS *Z*

The translation in this direction is possible to convert to previous case (translation in axes x and y). This situation is shown in Fig.3. We can calculate reduced distances b_{xr} and b_{yr} as

$$b_{xr} = b_x - b_z \frac{x_2'}{f_2} \tag{10}$$

$$b_{yr} = b_y - b_z \frac{y_2'}{f_2} \tag{11}$$

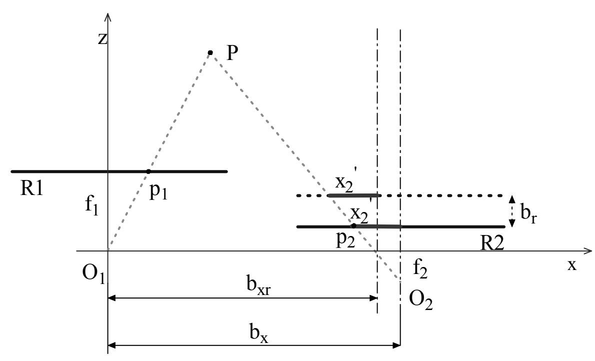

Fig. 3: *Translation in axis z*

This translation comes to previous case, if we use these reduced distances instead of distances b_x and b_y in formula (9).

2.3 ROTATION AROUND AXES *X* AND *Y*

Rotation around axis y is possible to convert to basic situation by the help of using coordinates $[x_{2r},y_{2r}]$ instead of coordinates $[x_2,y_2]$.

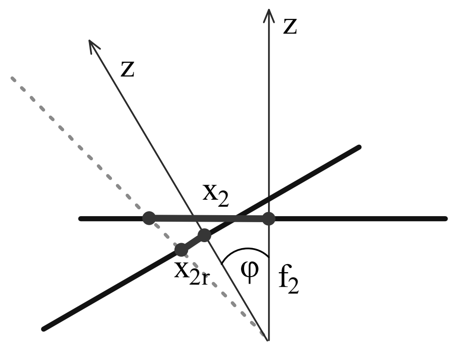

Fig. 4: *Rotation around axis y*

For reduced coordinate x_{2r} holds formula

$$x'_{2r} = f_2 \frac{x'_2 - f_2 tg\varphi}{f_2 + x'_2 tg\varphi}, \qquad (12)$$

similarly for rotation around axis x

$$y'_{2r} = f_2 \frac{y'_2 - f_2 tg\omega}{f_2 + y'_2 tg\omega}. \qquad (13)$$

2.4 ROTATION AROUND AXIS *Z*

Rotation around axis z we can recalculate to basic condition by using of rotation about angle κ. Situation is shown on Fig.5. If we have an original point with coordinates $[x_1,y_1]$ then rotated point about angle κ is calculated as

$$\begin{aligned} x_2 &= x_1 \cos\kappa - y_1 \sin\kappa \\ y_2 &= x_1 \sin\kappa + y_1 \cos\kappa \end{aligned} \qquad (14)$$

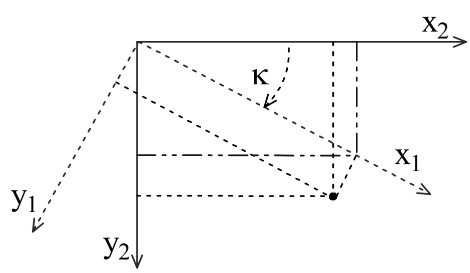

Fig. 5: *Rotation around axis z*

3 ALGORITHM OF WORLD COORDINATES CALCULATING

Situation at spatial point coordinates calculating is shown on Fig.2. We propose a point in front of cameras. On the left image the projection p_1 has coordinates $[x_1,y_1]$, projection p_2 on right image coordinates $[x_2',y_2']$. We assume calibrating cameras, parameters b_x, b_y, b_z, ω, φ, κ, f_1, f_2 are known.

Then the world coordinate is calculated by backward application of equations (9)-(14).

The algorithm is that:

1. from coordinates $[x_2',y_2']$ on right image is calculated reduced coordinates $[x_{2r}',y_{2r}']$ by a rotation about angle κ via formula (14).
2. from coordinates $[x_{2r}',y_{2r}']$ is calculated reduced coordinates $[x_{2r}',y_{2r}']$ by a rotation around axes x and y about angles ω and φ via formulas (12) and (13). Then only translations have left.
3. reduced lengths b_{xr} and b_{yr} are calculated from translation b_z via formulas (10) and (11). This way the translation in axis z is discarded and translations in axes x and y have only left.
4. spatial point coordinate z is calculated through the use of x_{2r}', y_{2r}', b_{xr} and b_{yr} via formula (9). Here we reflect an equation (2) in solving parallaxes p_x and p_y. Similarly coordinates x and y are calculated.

4 CONCLUSION

There was a tendency to describe a principle of 3D point reconstruction. An algorithm for a spatial point reconstruction in a case of free cameras rotation and different focal length was given. Corresponding pairs we can obtain for example by the help of correlation method. Correlations are computed by comparing a fixed window in the first image to a shifting window in the second. The second window is moved in the second image by integer increments along the corresponding epipolar line and a curve of correlation scores is generated for integer disparity values.

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