

# ANALYSIS OF MULTI-WIRE ANTENNAS WITH TIME-DOMAIN METHOD OF MOMENTS (TD-MOM)

Zbyněk LUKEŠ, Doctoral Degree Programme  
Dept. of Radio Electronics, FEEC, BUT  
E-mail: lukes@feec.vutbr.cz

Supervised by: Prof. Zbyněk Raida

## ABSTRACT

The paper deals with the wideband analysis of multi-wire antennas by the time-domain method of moments (TD-MoM). Two geometrical multi-wire structures are described: two linear parallel dipoles two-meter long with distance one meter between them, and two parallel dipoles with the distance 0.1 meter only. In these cases, we excite one dipole in the central segment. In the text, two types of solution are described. We use a quicker and more efficient method only. Our results are compared with commercial software in frequency domain (IE3D) from SWR point of view. Problems of solution are described in conclusion.

## 1 INTRODUCTION

Numerical analysis of wire antennas is based on integral formulation of Maxwell equations and on the exploitation of vector potentials [1]

$$\mathbf{A}(\mathbf{r}, s) = \iint_s \{G(\mathbf{r} | \mathbf{r}_0, s) \mathbf{J}(\mathbf{r}_0, s)\} dS \quad (1)$$

$$V(\mathbf{r}, s) = \iint_s \{G(\mathbf{r} | \mathbf{r}_0, s) \rho(\mathbf{r}_0, s)\} dS \quad (2)$$

Here,  $\mathbf{A}(\mathbf{r}, s)$  and  $V(\mathbf{r}, s)$  are vector potential and scalar one in the point described by the vector  $\mathbf{r}$ . If time-domain analysis is performed,  $s$  plays the role of time, if frequency domain analysis is done,  $s$  is the angular frequency. Next,  $G(\mathbf{r} | \mathbf{r}_0, s)$  is Green function describing contribution of a source (current, charge) in the point  $\mathbf{r}_0$  to the potential (vector one, scalar one) in the point  $\mathbf{r}$ . Finally,  $\mathbf{J}(\mathbf{r}_0, s)$  is current density on the antenna wire, and  $\rho(\mathbf{r}_0, s)$  is charge density there. The integration is performed over the antenna surface.

Current density and charge density are related by the continuity theorem

$$-\partial \rho(\mathbf{r}, s) / \partial t = \nabla \cdot \mathbf{J}(\mathbf{r}, s). \quad (3)$$

If vector and scalar potentials are known, we can compute electric intensity of the wave

$$\mathbf{E}^S(\mathbf{r}, s) = -\partial \mathbf{A}(\mathbf{r}, s) / \partial t - \nabla V(\mathbf{r}, s) \quad (4)$$

The set of equations (1) to (4) is solved by moment method: the antenna wire is divided into identical segments, the current on the segment is approximated by a constant function, and the residual is minimized by the collocation method.

### 1.1 MOMENT METHOD IN THE TIME DOMAIN

In the time domain, Green function is of the following form [2]:

$$G(\mathbf{r} | \mathbf{r}_0, t) = \frac{1}{4\pi |\mathbf{r} - \mathbf{r}_0|} \quad (5)$$

Phase shifts, which are given by different distances between sources  $\mathbf{r}_0$  and observation points  $\mathbf{r}$ , are represented by time delays in the representation of currents  $\mathbf{J}(\mathbf{r}, t - |\mathbf{r} - \mathbf{r}_0|/c)$  and charges  $\rho(\mathbf{r}, t - |\mathbf{r} - \mathbf{r}_0|/c)$ . Here,  $c$  denotes velocity of light.

Principle of the method is shown in **Fig. 1**. We excite the unknown system  $H(Z)$  by a known input quantity  $X(Z)$ , and then, we compute the response of the system  $Y(Z)$ . In our case, current distribution on the antenna wire plays the role of the response. Finally, we can compute parameters of the system

$$H(Z) = \frac{X(Z)}{Y(Z)} \quad (6)$$

Integral equations (1) to (4) are adapted for a current induced on a thin [2], finite-length perfect conducting cylindrical tube, which is excited by an incident electromagnetic field. The incident electric field is assumed to be invariant around the cylinder circumference and polarized along the length of the cylinder.

Consider a thin wire of length  $2h$  and radius  $a$  located symmetrically along  $z$ -axis as shown in the **Fig. 2**. The incident electric field induces current  $I(z, t)$ , which is a function of  $z$  and  $t$  only because of the thin-wire approximation. Using the reduced kernel approximation, the vector potential is given by

$$A_z(x, y, z, t) = \mu \int_{z'=-h}^h \frac{I(z', t - R/c)}{4\pi R} dz', \quad (7)$$

where

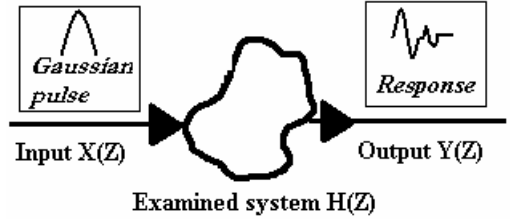
$$R = \sqrt{x^2 + y^2 + (z - z')^2 + a^2}, \quad (8)$$

and  $a$  is radius of the wire.

The total electric field is the sum of the incident and scattered fields. Next, we apply the boundary condition for total electric field, which implies that the  $z$  component of the total electric field must vanish on the conducting surface. Thus, we derive the electric field integral equation for a straight setting  $x = y = 0$ , in equation (8), and noting that

$$\nabla(\nabla \cdot A) = (\partial^2 A_z)/(\partial z^2),$$

given by



**Fig. 1:** Principle of method

$$\frac{\partial^2 A_z}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 A_z}{\partial t^2} = -\frac{1}{c^2} \frac{\partial E_z^i}{\partial t} \text{ for } z \in (-h, h), \quad (9)$$

where  $E_z^i$  is the  $z$  component of incident electric field  $E^i$ , and  $A_z$  is given by

$$A_z(z, t) = \mu \int_{z'=-h}^h \frac{I(z', t - \frac{|z-z'|}{c})}{4\pi \sqrt{|z-z'|^2 + a^2}} dz'. \quad (10)$$

## 2 NUMERICAL EXAMPLES

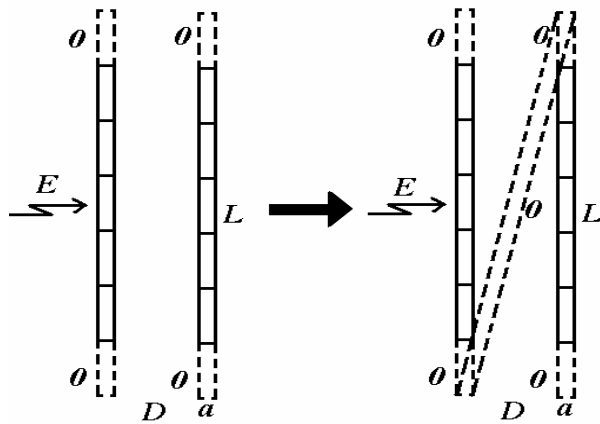
In this text, two geometrical structures are examined. First, two planar wire dipoles of the length  $L = 2$  m and of the wire radius  $a = 0.001$  m are analyzed. The distance between dipoles is  $D = 1$  m. Each dipole is divided into 30 equivalent segments. The antenna system is excited in the center of the first dipole (the segment number 16) by Gaussian plane wave [3]. Second, the distance between dipoles is change to  $D = 0.1$  m.

The analysis can be performed in two ways:

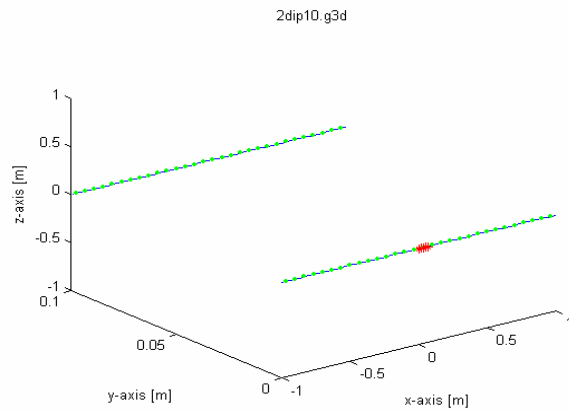
- We can extend the solution of (1) and (2) into 4 dimensions from the classical 3D space. The fourth dimension represents the number of separate wire lines.
- We solve only a 3D problem (only one 3D wire structure) by adding zero cells corresponding to zero values of potentials and current between wires.

Hardware requirements of the first method are enormous. The method was tested on a similar dipole only, and computation time has increased more than 40 times. The method is non-effective to analyzing the wire structure with very different number of cells (all rows and columns of matrix have to be filled by zero values everywhere the geometrical structure is not defined).

Using the second method, the problem can be solved only by adding three zero cells. The principle of this approach is shown in **Fig. 2**. In **Fig. 3**, the examined geometrical structure is depicted. The circles on the line represent segments centres, the star symbols show the source cell, where dipole is excited.



**Fig. 2** Principle of second case



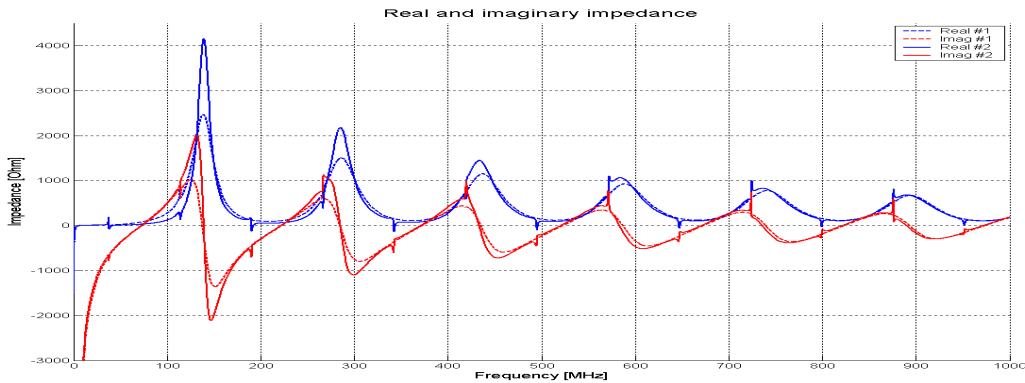
**Fig. 3** Geometrical structure of antennas

Using numerical computation of (1), (2) and (3), we obtain current distribution on the source cell. Using Fourier transformation of Gaussian pulse and current response [3], we can get the input impedance (see Fig. 4).

In **Tab. 1**, comparison of SWR values computed in time domain (Matlab algorithm) and in frequency domain (IE3D Zeland Software) is given. The table shows the first three resonances, where the antenna is matched.

Numerical Model #1 (D=1 m)						
	Matlab		IE3D		Error frequency [%]	Error SWR [%]
	MHz	SWR	MHz	SWR		
1. resonance	75	1.50	72	1.30	4.17	15.38
3. resonance	233	2.05	226	1.81	3.10	13.26
5. resonance	380	2.22	337	2.18	12.76	1.83
Numerical Model #2 (D=0.1 m)						
	Matlab		IE3D		Error frequency [%]	Error SWR [%]
	MHz	SWR	MHz	SWR		
1. resonance	75	1.70	75.4	2.80	0.53	39.29
3. resonance	228.5	1.18	221.2	2.32	3.30	49.14
5. resonance	382.5	1.60	369.6	1.51	3.49	5.96

**Tab. 1** Values of SWR computing in Matlab and IE3D



**Fig. 4** Spectrum of real and imaginary part of input impedance in range from 0 to 1000 MHz

### 3 CONCLUSION

The paper describes the numerical wideband analysis of multi-wire antennas. TD-MoM was used for the analysis. The program code was implemented in Matlab. The Matlab code was tested on two examples. The first antenna consisted of two parallel dipoles of the length  $L = 2$  m, diameter  $a = 2$  mm and the distance  $D = 1$  m. In the second case, the distance was changed to  $D = 0.1$  m.

The results of the analysis in Matlab were compared with results obtained by IE3D. For both methods, the SWR values were compared in frequency range from 0 to 1000 MHz. In **Tab. 1**, we can see the SWR values when antenna is matched. The percentage frequency error is very low (about 3 %). The SWR error is here about 1 to 50 %: these results are strongly influenced by the time step in Matlab computation and frequency step in IE3D solution

(frequency step was set to 2 MHz).

During the analysis, stability problems occurred (see **Fig. 4**). When changing the distance between wires from 1 m to 10 cm, in the frequency range, we can see the periodic oscillations on the frequencies depending on dipoles distance. The problem can be solved using the implicit form of solution [4].

## ACKNOWLEDGEMENT

Research described in the paper was financially supported by the research programs MSM 262200011 and MSM 262200022, by the grants of the Czech Grant Agency no. 102/03/H086 and 102/04/1079. Further financing was obtained via grant of the Grant Agency of Czech Ministry of Education no FRVŠ 1626/2004.

## REFERENCES

- [1] Černohorský, D., Raida, Z., Škvor, Z., Nováček, Z.: *Analýza a optimalizace mikrovlnných struktur (Analysis and Optimization of Microwave Structures)*. Brno: VUTIUM Publishing, 1999.
- [2] Rao, S. M.: *Time Domain Electromagnetics*. London: Academic Press, 1999.
- [3] Lukeš, Z., Raida, Z.: Analysis of wire dipole in frequency and time domain. In *Proceedings of the conference Radioelektronika 2003*. Brno: Brno Univ. of Technology, 2003, p. 237 – 240.
- [4] Raida, Z., Tkadlec, R., Franek, O., Motl, M., Láčík, J., Lukeš, Z., Škvor, Z.: *Analýza mikrovlnných struktur v časové oblasti (Time-domain analysis of microwave structures)*. Brno: VUTIUM Publishing, 2003. 232 pages. ISBN 8-0214-2541-5