PROPERTIES OF EXPLICIT AND IMPLICIT FORM SOLUTION BY TD-EFIE

Jaroslav LÁČÍK, Doctoral Degree Programme (1)
Dept. of Radio Electronics, FEEC, BUT
E–mail: xlacik00@stud.feec.vutbr.cz

Supervised by: Dr. Zbyněk RAIDA

ABSTRACT

The time-domain electric field integral equations (EFIE) are solved by the method of moments. The RWG functions are used as basis functions. The explicit and implicit forms are derived from TD-EFIE. Their properties are demonstrated on the example of a square plate, which is excited as a scatterer.

1 INTRODUCTION

The problem of obtaining transient response of an arbitrarily shaped conducting body excited either as an antenna or as a scatterer is the interest in electromagnetic community. Basically, there are two approaches to obtain the transient response of an arbitrarily shaped conducting body: 1) by obtaining the frequency response of the structure excited by time-harmonic sources and using the inverse Fourier transform techniques to calculate the required transient data [1] and 2) by formulating the problem directly in the time domain [2].

In this work, we consider the second approach. Two solution forms are presented, the explicit one [2] and the implicit [3] one. Their properties are demonstrated on the example of a conducting plate.

2 TIME-DOMAIN ELECTRIC FIELD INTEGRAL EQUATION FORMULATION

Let S denote the surface of a closed or open perfect electric conducting (PEC) body illuminated by a transient electromagnetic pulse. By enforcing the tangential electric field boundary condition on the PEC surface, the following integro–differential equation may be derived [2]

$$\left[\frac{\partial \mathbf{A}(\mathbf{r},t)}{\partial t} + \nabla \phi(\mathbf{r},t)\right]_{\tan} = \left[\mathbf{E}^{i}(\mathbf{r},t)\right]_{\tan}, \quad \mathbf{r} \in S, \tag{1a}$$

$$\left[\frac{\partial^2 \mathbf{A}(\mathbf{r},t)}{\partial t^2} + \nabla \psi(\mathbf{r},t)\right]_{\tan} = \left[\frac{\partial \mathbf{E}^{i}(\mathbf{r},t)}{\partial t}\right]_{\tan}, \quad \mathbf{r} \in S. \tag{1b}$$

The vector potential **A**, the scalar potential ϕ or the time derivative of the scalar potential ψ contain the unknown current density $\mathbf{J}(\mathbf{r}, t)$, which is induced by the incident electric field $\mathbf{E}^i$, **r** is an arbitrarily located observation point. The equation (1a) is used for the derivation of the implicit form [3], the time derivative of the vector potential is approximated by a backward finite difference. The equation (1b) is used for the derivation of the explicit form [2], the time derivative of the vector potential is approximated by a central finite difference. Thereafter, the equations (1a, 1b) are solved by the method of moments (MoM) [3]. The body is modeled with triangular patches. The unknown current density $\mathbf{J}(\mathbf{r}, t)$ is approximated as

$$\mathbf{J}(\mathbf{r},t) = \sum_{n=1}^{N} I_n(t)\mathbf{f}_n(\mathbf{r}), \tag{2}$$

where

$$\mathbf{f}_n = \begin{cases} \dfrac{l_n}{2A_n^{\pm}}\rho_n^{\pm}, & \mathbf{r} \in T_n^{\pm} \\ \mathbf{0}, & \textit{otherwise} \end{cases} \tag{3}$$

In (2), (3) $I_n(t)$ is the unknown coefficient representing the value of the component of the surface current normal to the nth edge, f_n is the basis function, l_n is the length of the edge, $A_n^{\pm}$ is the area of the triangle $T_n^{\pm}$ (Fig. 1), $\rho_n^{\pm}$ is the vector from resp. to the free vertex of $T_n^{\pm}$ and N is the number of nonboundary edges. A boundary edge is defined as an edge, which is associated with only one triangular patch. These basis functions are called RWG functions and their properties are described in [1].

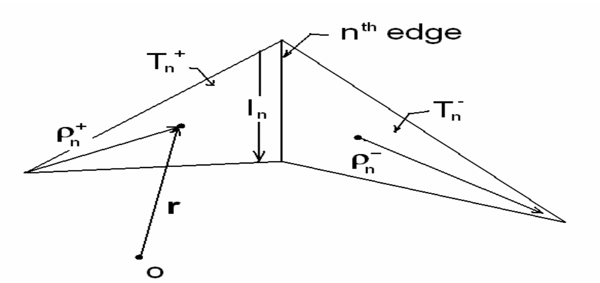

Fig. 1: *Triangle pair and their interior edge*

2.1 THE IMPLICIT FORM

After the testing procedure, it yields from (1a)

$$l_m\left(\frac{\rho_m^{c+}}{2}+\frac{\rho_m^{c-}}{2}\right)\cdot\frac{\mathbf{A}(\mathbf{r}_m,t_i)-\mathbf{A}(\mathbf{r}_m,t_{i-1})}{\Delta t} - l_m\left[\phi(\mathbf{r}_m^{c+},t_i)-\phi(\mathbf{r}_m^{c-},t_i)\right] = l_m\left(\frac{\boldsymbol{\rho}_\mathbf{m}^{\mathbf{c+}}}{2}+\frac{\boldsymbol{\rho}_\mathbf{m}^{\mathbf{c-}}}{2}\right)\cdot\mathbf{E}^i(\mathbf{r}_m,t_i),$$

$$m = 1,\ 2,\ 3\ldots\ N, \tag{4}$$

In (4) ρ_m^{c+} is the vector from the free vertex to the centroid of T_m^+ and ρ_m^{c-} is the vector from the centroid to the free vertex of T_m^-, $\mathbf{r}_m^{c\pm}$ resp. $\mathbf{r}_m$ represent the position vectors to the centroids of the triangle $T_m^{\pm}$ resp. to the center of mth edge.

The disadvantage of (4) is using the side finite difference and the numerical integration of the temporal current, which is necessary for computing the scalar potential ϕ [3].

2.2 THE EXPLICIT FORM

After the testing procedure, it yields from (1b)

$$l_m\left(\frac{\rho_m^{c+}}{2}+\frac{\rho_m^{c-}}{2}\right)\cdot\frac{\mathbf{A}(\mathbf{r}_m,t_i)-2\mathbf{A}(\mathbf{r}_m,t_{i-1})+\mathbf{A}(\mathbf{r}_m,t_{i-2})}{(\Delta t)^2}$$
$$-l_m\left[\psi(\mathbf{r}_m^{c+},t_{i-1})-\psi(\mathbf{r}_m^{c-},t_{i-1})\right]=l_m\left(\frac{\rho_m^{c+}}{2}+\frac{\rho_m^{c-}}{2}\right)\cdot\mathbf{F}^i(\mathbf{r_m},t_{i-1}),$$
$$m=1,\ 2,\ 3\ldots\ N, \tag{5}$$

where

$$\mathbf{F}^i=\frac{\partial\mathbf{E}^i}{\partial t}. \tag{6}$$

The meaning of symbols stays the same. We require [2] $\Delta t\le R_{\min}/c$, where $R_{\min}$ is the minimum distance between the edge centers and c is the velocity of propagation of the electromagnetic wave.

Using the central finite difference is the advantage of (5). This form is more accurate than the implicit form but the problem is the stability of solution.

3 NUMERICAL EXAMPLE

In this section, the numerical results are presented. The current is computed in the center of the square plate, which is excited as a scatterer by a Gaussian impulse plane wave of the form given by [3]

$$\mathbf{E}^i(\mathbf{r},t)=\mathbf{E}_0\frac{4}{T\sqrt{\pi}}e^{-[4/T(ct-ct_0-\mathbf{r}\cdot\mathbf{k}_0)]^2} \tag{7}$$

where $\mathbf{k}_0$ is the unit vector in the direction of propagation of the incident wave, T is the pulsewidth of the Gaussian impulse, $\mathbf{E}_0\cdot\mathbf{k}_0=0$ and t_0 is a time delay that represents the time at which the pulse peaks at the origin. The Gaussian impulse is with $\mathbf{E}_0=120\pi\mathbf{a}_\mathbf{x}$ V/m, $\mathbf{k}_0=-\mathbf{a}_z$, $T=2$ lm (ligthmeter), and $ct_0=6$ lm. Note that 1lm is the unit of time taken by the electromagnetic wave to propagate a distance of 1.0 m in free space. Comparisons are made with frequency domain data that were inverse Fourier transformed.

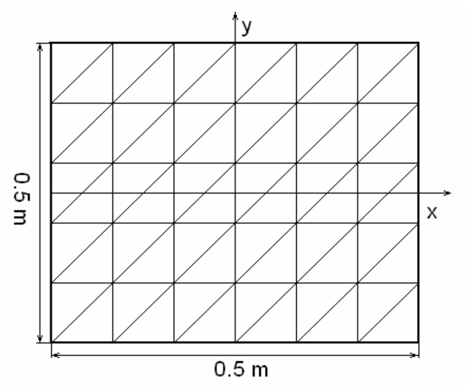

Fig. 2: *The analyzed square plate*

The square plate depicted in Fig. 2, *0.5 x 0.5* m, is located in the *xy* plane and centered about the origin. The plate is divided into six and five identical elements along *x* and *y* directions, resulting in 30 rectangular patches. This division allows us to obtain the current at the center of the plate directly.

The current response is shown on Fig. 3. The response of the explicit form agree very well with IDFT, but the late-time oscillations are in this one, which is the big problem of the explicit form. That instability depends on the shape of discretization patches and the choice of the time step [4]. The late-time oscillations can be put down with using the averaging technique [3], but it makes a little time shift in the results.

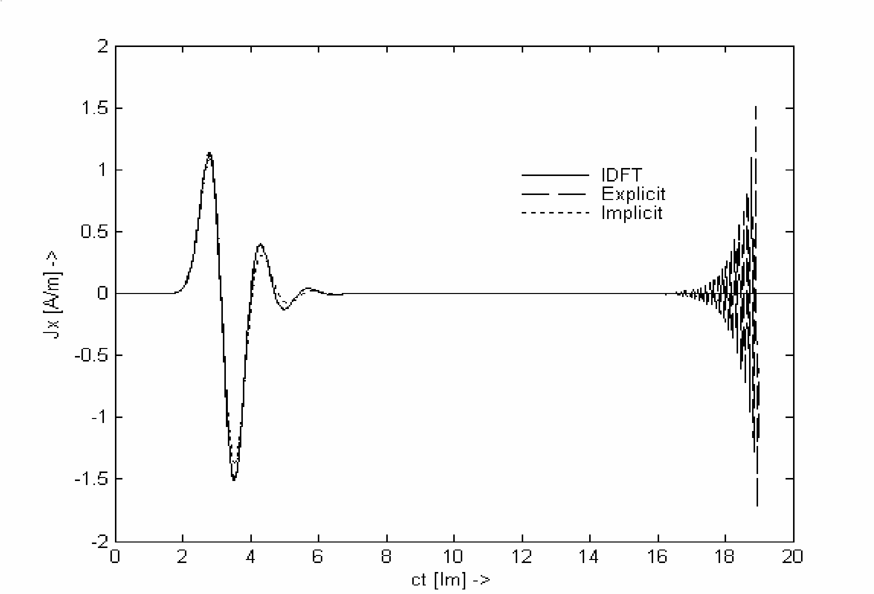

Fig. 3: *Transient current at the center of a conducting 0.5 m x 0.5 m plate located in the xy plane with 79 unknowns*

The response of the implicit form is less accurate with comparing IDFT (Fig.3), but the time late oscillations are not here. The less accuracy is caused by the side-finite difference in (4).

4 CONCLUSION

In this paper, the computing of the transient responses of antennas and scatterers is presented. The explicit and implicit forms are derived from TD-EFIE. Their properties are shown on the example of square plate, which is excited as a scatterer.

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REFERENCES

[1] Rao, S. M., Wilton, D. R., Glisson, A. W.: Electromagnetic scattering by surfaces of arbitrary shape. IEEE Transactions on Antennas and Propagation. 1982, vol. 30, no. 5, p. 409 – 418.

[2] Rao, S. M., Wilton, D. R.: Transient scattering by conducting surfaces of arbitrary shape. IEEE Transactions on Antennas and Propagation. 1991, vol. 39, no. 1, p. 56 to 61.

[3] Rao, S. M.: Time Domain Electromagnetics. London: Academic Press, 1999.

[4] Manara, G., Monorchio, A., Reggiannini, R.: A Space–time discretization criterion for a stable time–marching solution of the electric field integral equation. IEEE Transactions on Antennas and Propagation. 1997, vol. 45, no. 3, p. 527 – 532.