DENOISING ECG SIGNALS USING WAVELET TRANSFORM

Lukáš CHMELKA, Doctoral Degree Programme (1)
Dept. of Biomedical engineering, FEEC, BUT
E-mail: xchmel06@stud.feec.vutbr.cz

Supervised by: Dr. Jiří Kozumplík

ABSTRACT

Wavelet transform is well known method of decomposing signals. This paper will show that is useful for denoising nonstacionarity signals eg. the ECG signals.

1 INTRODUCTION

Wavelet transform was realized with support of Matlab and Wavelet Toolbox [1] who can make some wavelet filters banks eg. Daubechies (`db5, db6`) and biorthogonal (`bior2.8,bior3.9`) and others. Designed wavelet filtering was tested on many ECGs from Cammon Standard in Electrocardiography (CSE) library.

2 PROCESSING THROUGH THE WAVELET TRANSFORM

Denoising the ECG signals through the wavelet transform is shortly described in three sections. For first has the signal decomposed to a few frequency bands, modifying wavelet coefficients and reconstruction.

2.1 WAVELET DECOMPOSITION

It exists many ways to obtain the wavelet coefficients [2]. It has been compared two methods: the well known clasical structure of discrete wavelet transform with decimation and the same without the decimation (redundant or shift-invariant). On Figure 1 is shown block diagram of the second method where $\mathrm{H}(z)$ and $\mathrm{H_r}(z)$ are decomposition and reconstruction highpass filters. The $\mathrm{G}(z)$ and $\mathrm{G_r}(z)$ are lowpass filters. The $d(\cdot,\cdot)$ are decomposition coefficients and $a(\cdot,\cdot)$ are approximation coefficients.

For determination the better method to denoising were mentioned methods put to experiment. Input was a signal (*Doppler* generated with command `wnoise`[1]) with an

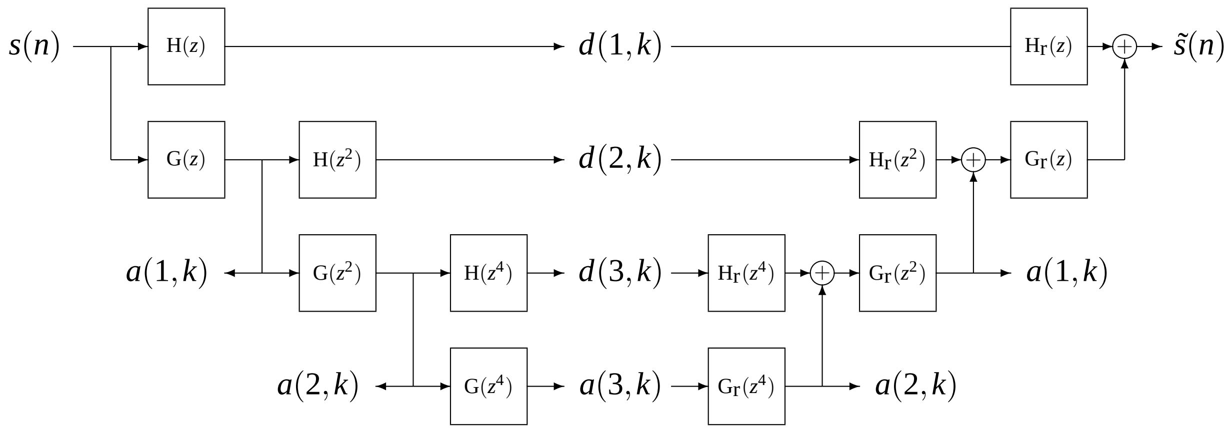

Figure 1: Shift invariant decomposition and reconstruction method

additive noise. Signal to noise ratio (SNR) was measured SNR = 10 dB by following relation:

$$\mathrm{SNR}_{\mathrm{dB}} = 10\log\frac{\sum_{n=0}^{N-1} s(n)^2}{\sum_{n=0}^{N-1} v(n)^2} \approx 10\log\frac{\sum_{n=0}^{N-1} s(n)^2}{\sum_{n=0}^{N-1}\left(s(n)-\tilde{s}(n)\right)^2}, \tag{1}$$

where $s(n)$ is a clean signal (without noise), $v(n)$ is a noise and $\tilde{s}(n)$ is an estimation of $s(n)$.

This signal was decomposed of both structures to 6 levels. On Figure 1 is demonstrated decomposition of the signal to 3 levels. For decomposition were used different decomposition and reconstruction bank of filters eg. Daubechies (`db5,db6,db7`) and biorthogonal (`bior2.8, bior3.7, bior3.9` and `bior6.8`)[1].

In each level was looked for optimum size of threshold (see below) to get maximum SNR or minimum of Mean Square Error (MSE)

$$\mathrm{MSE} = \mathbb{E}\left\{\left(s(n)-\tilde{s}(n)\right)^2\right\} \approx \frac{1}{N}\sum_{n=0}^{N-1}\left(s(n)-\tilde{s}(n)\right)^2. \tag{2}$$

For minimize function MSE and determination responsible sizes of the thresholds was used Matlab generated function `fminsearch`. Results has been organized to the table.

	Shift-invariant		**Transf. with decimation**	
N	**Filter**	**SNR [dB]**	**Filter**	**SNR [dB]**
18	db5	18.4	db5	16.8
22	db6	18.6	db6	17.3
26	db7	18.7	db7	16.2
18	bior2.8	17.2	bior2.8	16.3
18	bior3.7	17.8	bior3.7	15.9
22	bior3.9	17.7	bior3.9	16.2
26	bior6.8	19.2	bior6.8	17.8

Table 1: Comparison of two mentioned decomposition methods

It can be seen that shift-invariant method of decomposition give better results in issue of denoising.

2.2 SELECTION OF ONE DECOMPOSING FILTERS BANK

Is evident, from previous Table 1, that the SNR has been depend on kind of bank of decomposition filters. It isn't exist the algorithm for choice the right bank of filters for given problem. In this fact were compared wavelet coefficients obtained by decomposition with different bank of filters. A few of compared decomposition coefficients can be seen on Figure 2. Input signal was a clean ECG with additive noise, SNR was approx 12dB.

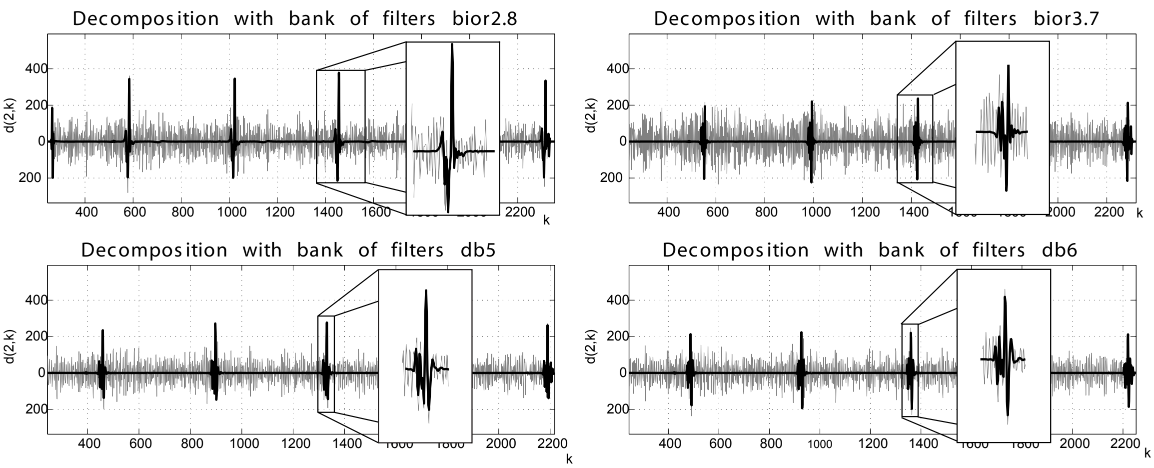

Figure 2: Some of compared decomposition coefficient

On the Figure 2 is shown two signals. The gray is wavelet coefficient of mentioned signal with noise and the black is wavelet coefficient of clean ECG. On the same Figure 2 are thin and tall impulses which are a component of QRS complex. Those wavelet coefficients are thresholded (see below) and resultant signal after denoising has had a smaller size of R-wave in comparation with input signal. For thresholding are fitting wavelet coefficients which has mentioned component of QRS complex much higher then noise around. For denoising the ECG is suitable decomposition with shift-invariant method by biorthogonal bank of filters `bior2.8`.

2.3 THRESHOLDING THE WAVELET COEFFICIENTS

Wavelet coefficients are modifying by Thresholding. In case of denoising is used soft thresholding it means that every coefficient which is higher then threshold is decreased by size of threshold. These coefficients which are under the threshold are deleted.

Size of the threshold λ can be determined as universal thresholdby the relation [3]:

$$\lambda = \sigma\sqrt{2\log N}, \tag{3}$$

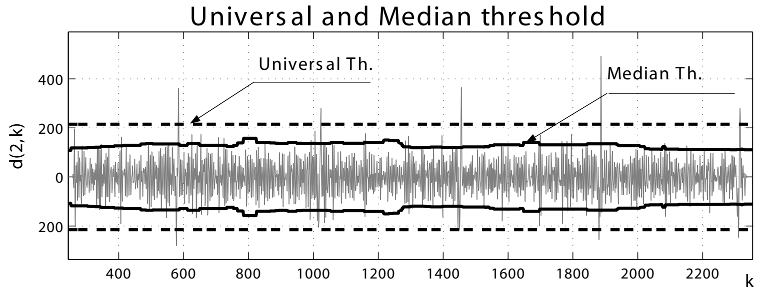

Figure 3: Universal and adaptive threshold

where σ is standard deviation and N is length of signal in each frequency level. This threshold is too high. Better is to use adaptive threshold obtained by order statistics filter [4]. On the Figure 3 is shown second level wavelet coefficients, computed universal and adaptive threshold.

3 RESULTS

This designed method was compared with filtering by filter design using the window method. Input signal was decomposed to 5 levels and thresholds were computed as adaptive.

In following figure are presented filtrated signal by designed method and by low pass FIR filter ($\omega_m \approx 40\,\text{Hz}$). Signal to noise ratio was computed for wavelet filtering

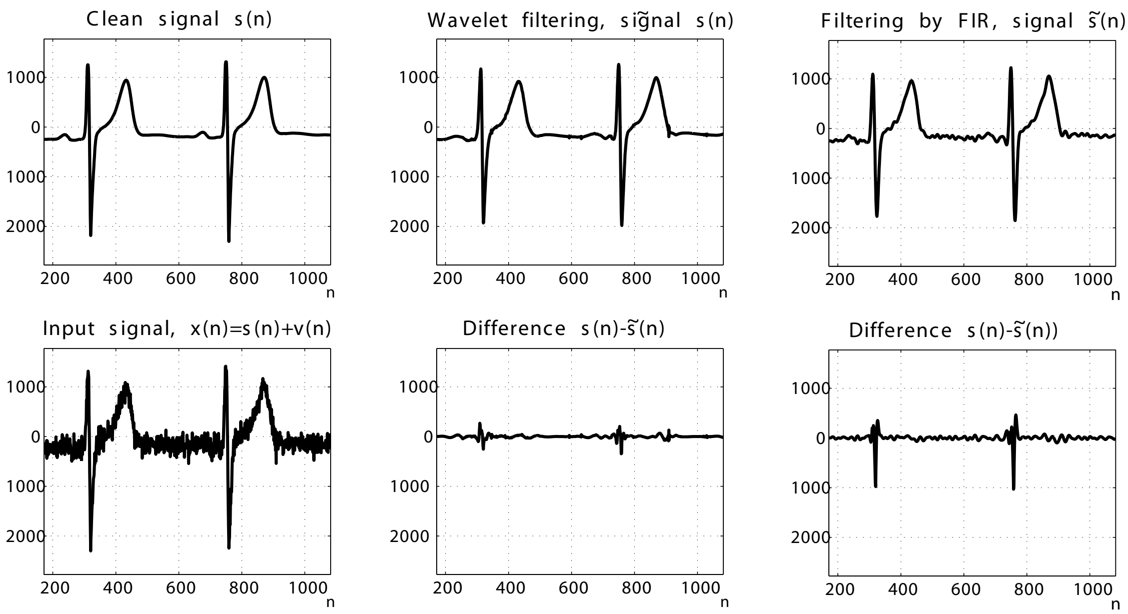

Figure 4: Filtration by two mentioned ways

SNR = 21.7 dB, for FIR filtering SNR = 13.2 dB when SNR of the input signal was SNR = 12.4 dB.

4 CONCLUSIONS

Filtration was applied for many ECG signals and in every cases was the wavelet filtering better then FIR filter. The reason is that spectrum of the noise interfere with spectrum of the ECG signal. By wavelet filtering are filtrated some frequency levels independent each other, whereas by classical filtration isn't possible to separate the signal and noise. Therefore is using wavelet filtering more useful then FIR filtering.

ACKNOWLEDGEMENT

The paper has been prepared as a part of the solution of GAČR project No. 102/02/0472 and with the support of the research plan No. J22/98:262200011.

REFERENCES

[1] Misiti, M. and Y., Oppenhaim, G., Poggi, J. M.: MATLAB Wavelet toolbox user's guide, The Math-Works Inc. 1996

[2] Daubechies, I.,: Ten Lectures on Wavelets. Society of Industrial and Appiled Mathematics, Pensylvania, 1992

[3] Donoho, D., L.: De-noising by Soft-Thresholding. IEEE Trans. Inform. Theory 41, 1995, pp. 613–627

[4] Jan, J.,: Digital Signal Filtering, Analysis and Restoration. VUTIUM Press, Technical University of Brno, 2000