USAGE OF STATISTICAL PROCESS CONTROL FOR MEASUREMENT

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ABSTRACT

It has to be ensured system measurement integrity for the valid results of the each test by the aid of periodic calibration. In the cases, when the procedure of the calibration can't rely on external testing measuring equipments or standards, statistical measurement process control provides problem solution. This thesis describes its methodology.

1 INTRODUCTION

In traditional SPC applications, the monitoring of testing or calibrating processes is done by using process control limits. Process control limits consist of performance specifications expanded to include measurement process uncertainty contributions. These contributions are arrived by multiplying measurement process uncertainties by statistical confidence multipliers, which are determined in accordance with the degree of confidence desired by monitoring the process. Measured values are plotted against these control limits. The appearance of value out of limits on a control chart is taken to indicate a process out of control, possibly a measuring device operating out of tolerance. Identifying the cause of measurement out of control often requires human analysis, but it is difficult for usage in special situations or inaccessible environments.

Statistical measurement process control (SMPC) methods can deal with previous problem situations. With SMPC, as with traditional SPC methods, the results of measurements are used to develop information regarding the accuracy of the monitoring process. This information takes the form of in-tolerance probabilities and bias (error or offset) estimates for measuring attributes. In-tolerance probabilities can be used to indicate instances where monitoring devices should be either taken out of service or derated. Bias estimates can be used as error correction factors to be applied to subsequent measurements.

2 METHODOLOGY OF SMPC

SMPC can be used to estimate in-tolerance probabilities and biases for both testing measuring equipments (TME) and standards. Solving for in-tolerance probability estimates involves finding statistical probability density function (*pdf*) for the quantities of interest and calculating the chances that these quantities will lie within their tolerance limits. If *f(x)* represents the *pdf* for a variable *x*, and *+L* and *-L* represent its tolerance limits, then the probability, that *x* is in-tolerance, is obtained by integrating *f(x)* over [*-L*, *L*]:

$$P = \int_{-L}^{L} f(x)dx \qquad (1)$$

This method can be used for the next situation.

There are three instruments with identical tolerances of ±10 units. One instrument measures an unknown quantity as 0 units; the second measures +6 units and the third measures +15 units. According to the first instrument, the third one is out-of-tolerance. According to the third instrument, the first one is out-of-tolerance. Which is out-of-tolerance?

The measurement configuration is shown in Fig. 1 and tabulated in column 1 of Tab. 1. Let instrument 1 act the role of a unit-under-test (UUT) and label its indicated value as Y, the "0" subscript labels the UUT. Likewise, let instruments 2 and 3 function as TME, label their declared values as Y_1, and Y_2 respectively, the "1" and "2" subscripts label TMEl and TME2 and define the variables

$$X_1 = Y_0 - Y_1 = -6,$$

and

$$X_2 = Y_0 - Y_2 = -15.$$

These quantities can be used to solve for the UUT (Instrument 1) in-tolerance probability estimate.

UUT = TME1	**UUT = TME 2**	**UUT = TME 3**
$L_0 = 10$	$L_0^{/} = 10$	$L_0^{//} = 10$
$L_1 = 10$	$L_1^{/} = 10$	$L_1^{//} = 10$
$L_2 = 10$	$L_2^{/} = 10$	$L_2^{//} = 10$
$Y_0 = 0$	$Y_0^{/} = 6$	$Y_0^{//} = 15$
$Y_1 = 6$	$Y_1^{/} = 0$	$Y_1^{//} = 6$
$Y_2 = 15$	$Y_2^{/} = 15$	$Y_2^{//} = 0$
$X_1 = -6$	$X_1^{/} = 6$	$X_1^{//} = 9$
$X_2 = -15$	$X_2^{/} = -9$	$X_2^{//} = 15$

Tab. 1: *Acquired results arranged for SMPC analysis*

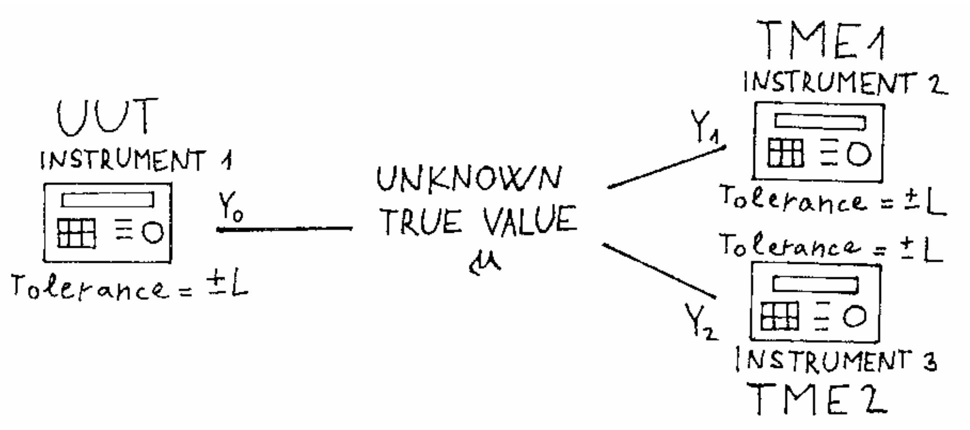

Fig. 1: *UUT in-tolerance estimate*

Solving for the in-tolerance probability of instrument 1: the notation $P(\omega|x)$ is used to denote the probability, that an event ω will occur, given that an event x has occurred. Here, ω may represent the event, that a UUT attribute is in tolerance and x may represent the event, that was obtained a set of measurements X_1 and X_2 of the attribute‘s value. In this case, $P(\omega|x)$ is the probability, that the UUT attribute is in-tolerance, given that the measurements results X_1 and X_2 were obtained. $P(\omega|x)$ is a conditional probability.

It can be also form conditional *pdfs*. It can be form a conditional *pdf* for a attribute error ε being present given, that it were obtained the quantities X_1 and X_2. With $f(\varepsilon|X_1, X_2)$ can be estimated an in-tolerance probability for instrument 1 by using it as the pdf in Eq. (1).

Following this procedure yields an in-tolerance probability estimate of approximately 77%.

Solving for the in-tolerance probability of instruments 2 and 3: it becomes apparent, that there is nothing special about instrument 1, that should motivate calling it the UUT. Likewise, there is nothing special about instruments 2 and 3, that should brand them as TME. Alternatively, instrument 2 could have been labeled the UUT and instruments 1 and 3 the TME, as in Fig. 2 and column 2 of Tab. 1.

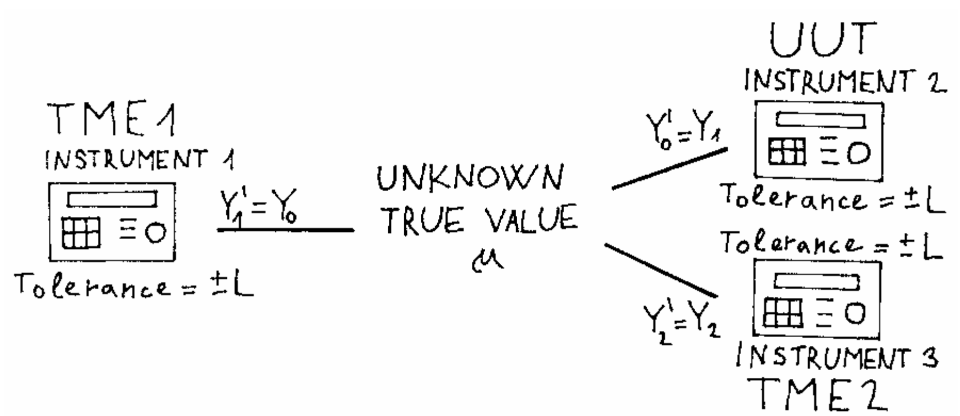

Fig. 2: *Exchanging UUT and TME roles*

This rearrangement of labels allows to calculate the in-tolerance probability for instrument 2 just as it has done for instrument 1. This involves defining the quantities

$$X_1^{/} = Y_0^{/} - Y_1^{/} = +6$$

and

$$X_2^{/} = Y_0^{/} - Y_2^{/} = -9,$$

and forming the pdf $f(\varepsilon| X_1^{/}, X_2^{/})$. Using the pdf in Eq. (1) yields an in-tolerance probability estimate of 99 % for instrument 2.

Similarly, if is computed

$$X_1^{//} = Y_0^{//} - Y_1^{//} = +9$$

and

$$X_2^{//} = Y_0^{//} - Y_2^{//} = +15,$$

construct the pdf $f(\varepsilon| X_1^{//}, X_2^{//})$ and used this pdf in Eq. (1), is acquired an in-tolerance probability estimate of 69 % for instrument 3.

Solving for instrument biases: bias or error of an attribute can be found by solving for the attribute‘s expectation value. This expectation value is equal to the attribute‘s mean value. The mean value is obtained by multiplying the attribute‘s conditional pdf by the error and integrating over all. With this prescription, the biases of instruments 1, 2 and 3 are given by

$$\text{Instrument 1 bias} = \int_{-\infty}^{\infty} \varepsilon f(\varepsilon \mid X_1, X_2) d\varepsilon,$$

$$\text{Instrument 2 bias} = \int_{-\infty}^{\infty} \varepsilon f(\varepsilon \mid X_1^{/}, X_2^{/}) d\varepsilon$$

and

$$\text{Instrument 3 bias} = \int_{-\infty}^{\infty} \varepsilon f(\varepsilon \mid X_1^{//}, X_2^{//}) d\varepsilon \qquad (2)$$

Using Eq. (2), the biases of instruments 1, 2, and 3 are estimated to be -7, -1 and +8.

The bias estimates can be employed as measuring attribute correction factors.

Now it can be answered question, which instrument is out-of-tolerance. Instrument 1 has an estimated bias of -7 and an in-tolerance probability of 77%, instrument 2 has an estimated bias of -1 and an in-tolerance probability of 99% and instrument 3 has an estimated bias of +8 and an in-tolerance probability of 69%. General purpose test equipment is usually managed to an end-of-period measurement reliability target of 72%. Accordingly, the decision to ensue from these results would be to submit instrument 3 to a higher level facility for recalibration.

Before placing too much stock in the above bias estimates, it has to be consider, that their computed 95% confidence limits are fairly wide:

Instrument 1: -13,4 to -0,6

Instrument 2: -7,4 to +5,4

Instrument 3: + 1,6 to +14,4

The wide ranges are due to the wide spread of the measured values and to the fact, that all instruments were considered with an equal accuracy.

Computing attribute correction factors

Suppose, that the Instrument 1 is a monitoring system, and instruments 2 and 3 are subject items. Then, following measurements of the attributes of Instruments 2 and 3 by the measuring system and application of SMPC, the monitoring system attribute could be assigned a correction factor β, which would be calculated using appropriate pdfs as shown in Eq. (2). The attribute could be compensated or corrected for "in software" by automatically subtracting the value β from subsequent monitoring system measurements.

Accommodation of Check Standards

In applying SMPC with a check standard, the check standard merely takes on the role of an additional subject item, albeit a comparatively accurate one. By using check standards, not only can the in-tolerance probabilities and biases of the attributes of monitoring systems be more accurately estimated, but in-tolerance probability and bias estimates can also be determined for the check standards.

3 CONCLUSION

With the help of statistical measurement process control methods can be acquired data which provide information to correction of known errors, give information when to recalibrate and when is the measurement process out of control or headed there and when take corrective action.

LITERATURE

[1] Hruška, K., Bradík, J.: Stanovení nejistot při měření parametrů jakosti, Brno, VUT v Brně 2001, ISBN 80-214-1656-1