LIGHT PROPAGATION IN THICK MAGNETO-OPTIC LAYERS

Ondřej ŽIVOTSKÝ, Master Degree Programme (5)
Dept. of Electronics and Telecommunication Technics, FEI, VŠB-TU Ostrava
E-mail: ondrej.zivotsky@post.cz

Supervised by: Dr. Mgr. Kamil Postava

ABSTRACT

A general matrix method for description of light reflection and transmission in systems containing of thin and thick anisotropic layers is proposed. The non-coherent summation of intensities in a thick layer is characterized using the coherence vector formalism. The measured quantities (magneto-optical rotation and ellipticity) are obtained from the 4 x 4 matrix describing transformation between coherence vectors, hereafter the coherence transforming matrix. Numerical simulations are shown in the case of polar magneto-optical geometry, in which the matrices have diagonal form.

1 INTRODUCTION

Fast development in the field of magneto-optical recording and information systems has resulted in the realization of matrix method for the description of light reflection and transmission by thin and thick magneto-optic films. In the case of thin films, [1-4] the interference effects are described using coherent summations of Jones matrices (the optical zig-zag path exceeds the coherence length of incident light: $2\, k_{zj}\, d \ll k_0\, l_c$, where $k_0 = 2\pi / \lambda$, d denotes the layer thickness, and l_c is the coherence length). This condition is broken for instance in the case of thick transparent substrate, in which no interference can be observed. This is the case of systems used for magneto-optical recording, in which light is reflected from the second face of thick substrates.

This article is divided into three parts. In section 2, the total forward reflection and transmission coherence transforming matrices are obtained by use of recurrence formulae based on matrix summation. In this section we propose the generalization of this method for multilayer systems consisting of thin and thick layers. In section 3, a few examples of numerical simulations are shown.

2 THIN AND THICK MAGNETO-OPTIC LAYERS

This section describes the incoherent intensity summation in a thick magneto-optic layer on the basis of the coherence matrix formalism. Let the layer under consideration be thick and

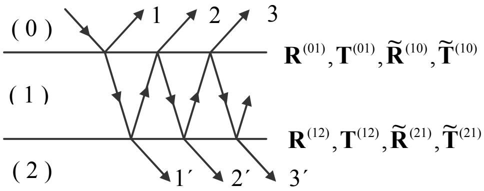
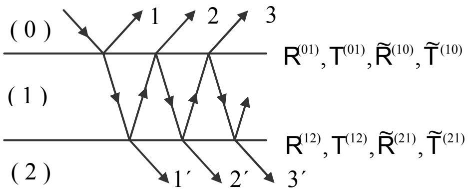
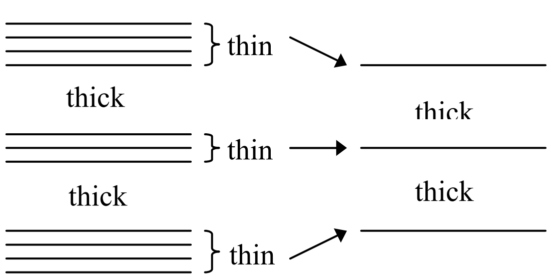
	Thin anisotropic layer	**Thick weakly anisotropic layer**
Example		
Conditions	thin: $l_c \gg 2\frac{k_{zj}}{k_0} d = 2N_z d$	thick: $l_c \ll 2\frac{k_{zj}}{k_0} d = 2N_z d$ weakly anisotropic: $l_c \gg 2\frac{\Delta k_{zj}}{k_0} d$
Description	Jones matrix: $\mathbf{R} = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix}$	Transforming coherence matrix: $\mathsf{R} = \begin{bmatrix} \mathsf{R}_{11} & \mathsf{R}_{12} & \mathsf{R}_{13} & \mathsf{R}_{14} \\ \mathsf{R}_{21} & \mathsf{R}_{22} & \mathsf{R}_{23} & \mathsf{R}_{24} \\ \mathsf{R}_{31} & \mathsf{R}_{32} & \mathsf{R}_{33} & \mathsf{R}_{34} \\ \mathsf{R}_{41} & \mathsf{R}_{42} & \mathsf{R}_{43} & \mathsf{R}_{44} \end{bmatrix}$
Reflection matrix	$\mathbf{R} = \mathbf{R}^{(01)} + \tilde{\mathbf{T}}^{(10)}\tilde{\mathbf{P}}_{24}\mathbf{R}^{(12)}\mathbf{P}_{13}\times$ $\times(\mathbf{I} - \tilde{\mathbf{R}}^{(10)}\tilde{\mathbf{P}}_{24}\mathbf{R}^{(12)}\mathbf{P}_{13})^{-1}\,\mathbf{T}^{(01)}$	$\mathsf{R} = \mathsf{R}^{(01)} + \tilde{\mathsf{T}}^{(10)}\tilde{\mathsf{P}}_{24}\mathsf{R}^{(12)}\mathsf{P}_{13}\times$ $\times(\mathsf{I} - \tilde{\mathsf{R}}^{(10)}\tilde{\mathsf{P}}_{24}\mathsf{R}^{(12)}\mathsf{P}_{13})^{-1}\,\mathsf{T}^{(01)}$
Transmission matrix	$\mathbf{T} = \mathbf{T}^{(12)}\mathbf{P}_{13}(\mathbf{I} - \tilde{\mathbf{R}}^{(10)}\tilde{\mathbf{P}}_{24}\mathbf{R}^{(12)}\mathbf{P}_{13})^{-1}\,\mathbf{T}^{(01)}$	$\mathsf{T} = \mathsf{T}^{(12)}\mathsf{P}_{13}(\mathsf{I} - \tilde{\mathsf{R}}^{(10)}\tilde{\mathsf{P}}_{24}\mathsf{R}^{(12)}\mathsf{P}_{13})^{-1}\,\mathsf{T}^{(01)}$

Table 1: *Description of thin and thick layers*

weakly anisotropic. The layer is thick, if partial waves are incoherent and do not interfere, i.e., the optical zig-zag path exceeds the coherence length of incident light: $2\ k_{zj}\ d \gg k_0\ l_c$. The weak anisotropy is described by condition $2\ \Delta k_z\ d \ll k_0\ l_c$, where Δk_z denotes the difference of the optical paths of the forward or backward waves in the anisotropic layer.Table1 compares the matrix description of light propagation in thin and thick magneto-optic layers. Partial waves reflected and transmitted by a layer after several zig-zag propagations are shown. The total reflected and transmitted light is an infinite sum of the contributions from the partial reflected and transmitted waves 1, 2, 3, … and 1′, 2′, 3′, …, respectively.

The elements of the total coherence transforming matrices R and T relate to observable ellipsometric and magneto-optical quantities characterizing reflection and transmission properties of a thick anisotropic layer, respectively. For example, the reflectivity R, the transmittance T, the ellipsometric angles θ_K and ε_K, or the degree of polarization P for polar magneto-optical geometry are given in the form: [5, 6]

$$R_p = \mathsf{R}_{11},\quad R_s = \mathsf{R}_{44},\quad R = \left(R_p + R_s\right)/2, \tag{1}$$



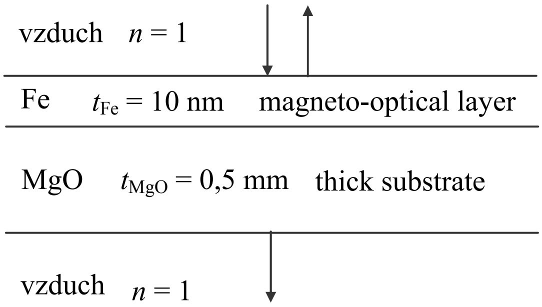


Fig. 1: *Recurrence for multilayer system*

Fig. 2: *Structure and basic optical and consisting of thin and thick layers magneto-optical parametres of studying system*

$$T_p = \mathcal{T}_{11}, \;\; T_s = \mathcal{T}_{44}, \;\; T = \left(T_p + T_s\right)/2, \tag{2}$$

$$P = \frac{\sqrt{\left(\mathcal{R}_{11} - \mathcal{R}_{44}\right)^2 + 4\mathcal{R}_{22}\,\mathcal{R}_{33}}}{\mathcal{R}_{11} + \mathcal{R}_{44}}, \tag{3}$$

$$\theta_K = -\frac{i}{2}\frac{\mathcal{R}_{22} - \mathcal{R}_{33}}{\mathcal{R}_{22} + \mathcal{R}_{33}}, \qquad \varepsilon_K = -\frac{1}{2}\frac{\mathcal{R}_{11} - \mathcal{R}_{44}}{\mathcal{R}_{22} + \mathcal{R}_{33}}. \tag{4}$$

The matrix method devoted to light reflection and transmission by a single layer can be applied recurrently. Figure 1 shows the generalization of the method for multilayer systems consisting of thin and thick layers. Calculations of the quantities characterizing the multilayer system is performed in four steps: [7]

- Calculation of reflection and transmission coefficients for all interfaces and thin-film subsystems [4] (see Table 1).
- Transformation of Jones matrices of the thin-film subsystem to coherence transforming matrices using equtation:

$$\mathcal{R} = \mathrm{R} \otimes \mathrm{R}^* = \begin{bmatrix} R_{11}\,R_{11}^* & R_{11}\,R_{12}^* & R_{12}\,R_{11}^* & R_{12}\,R_{12}^* \\ R_{11}\,R_{21}^* & R_{11}\,R_{22}^* & R_{12}\,R_{21}^* & R_{12}\,R_{22}^* \\ R_{21}\,R_{11}^* & R_{21}\,R_{12}^* & R_{22}\,R_{11}^* & R_{22}\,R_{12}^* \\ R_{21}\,R_{21}^* & R_{21}\,R_{22}^* & R_{22}\,R_{21}^* & R_{22}\,R_{22}^* \end{bmatrix} = \begin{bmatrix} \mathcal{R}_{11} & \mathcal{R}_{12} & \mathcal{R}_{13} & \mathcal{R}_{14} \\ \mathcal{R}_{21} & \mathcal{R}_{22} & \mathcal{R}_{23} & \mathcal{R}_{24} \\ \mathcal{R}_{31} & \mathcal{R}_{32} & \mathcal{R}_{33} & \mathcal{R}_{34} \\ \mathcal{R}_{41} & \mathcal{R}_{42} & \mathcal{R}_{43} & \mathcal{R}_{44} \end{bmatrix}. \tag{5}$$

- Calculation of the reflection and transmission coherence matrices R, Tof the total system (see Table 1).

- The observable quantities are obtained from total R, T matrices, in polar magneto-optical geometry are using Eqs. (1-4).

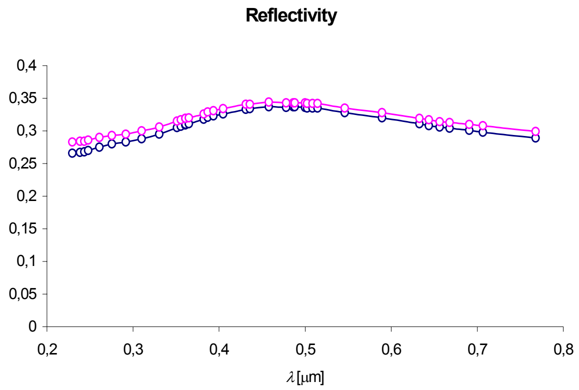


Fig. 3: *Relation between reflectivity R and wavelength λ*

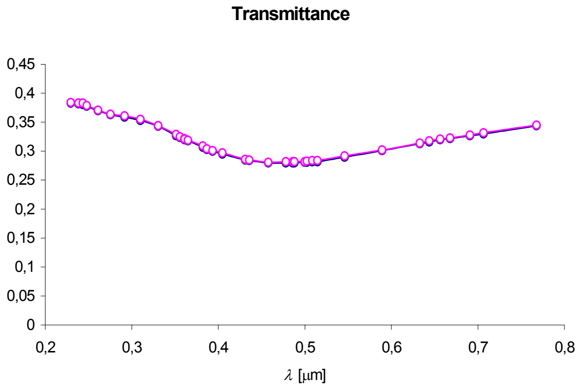


Fig. 4: *Relation between transmittance T and wavelength λ*

3 NUMERICAL SIMULATIONS

In this section, we use proposed matrix method for the special system, whose structure is shown in Figure 2. Numerical simulations were provided in polar magneto-optical geometry at normal incidence similarly as for magneto-optic recording applications.

The results of numerical simulations using Eqs. (1-4) are shown in Figs. 3-6. Simulation of each parameter was provided in two cases:

- including the reflection from the second face of thick substrate using method described in this paper - pink line
- without including the reflection from the second face of thick substrate - blue line

Optical and magneto-optical constants for MgO and Fe were obtained from References [8,9].

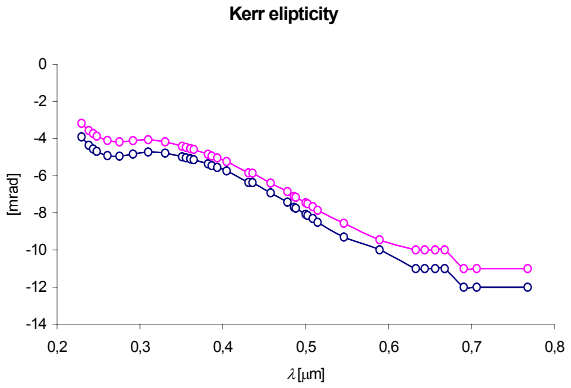


Fig. 5: *Relation between Kerr ellipticity ε_K and wavelength λ*

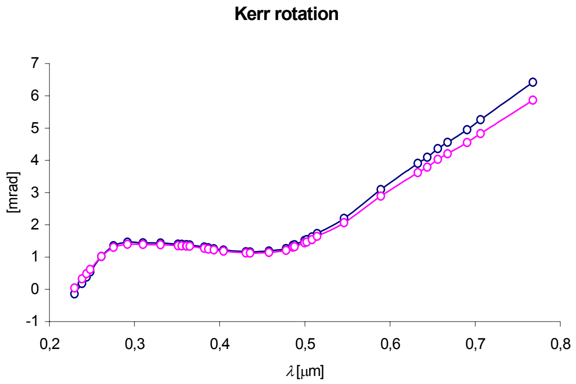


Fig. 6: *Relation between Kerr rotation θ_K and wavelength λ*

4 CONCLUSION

The matrix approach based on partial wave summation was developed for description of light reflection and transmission by thin and thick anisotropic layers. The coherence transforming matrix formalism was applied for description partial wave addition in a thick anisotropic films and compact formulae were obtained. Although these matrix formulae are suitable rather for numerical calculations, the analytic solution could be obtained in the special non-conversion case. Numerical simulations, present in section 3, describe the typical situation in magneto-optical recording.

ACKNOWLEDGEMENT

The research has been partially supported by the Grant Agency of Czech Republic (Grant No. 105/01/0168 and No. 202/01/0077).

REFERENCES

[1] R. M. A. Azzam and N. M. Bashara, Ellipsometry and Polarizes Light, 2nd ed. (North-Holland, Amsterdam 1987).

[2] I. Ohlídal and D. Franta, Ellipsometry of thin film systems, Progress in Optics, E. Wolf, ed. (North-Holland, Amsterdam 2000), Vol.41.

[3] K. Postava and O. Životský, Optical spectroscopy of dielectric layers for ULSI, Fine mechanics and optics, 54-58 (2003), Vol. 48, in Czech.

[4] Š. Višňovský, Magneto-optical ellipsometry, Czech. J. Phys. B 36 625-650 (1986).

[5] O. Životský, K. Postava, M. Foldyna, T. Yamaguchi, J. Pištora, Magneto-optics of systems containing non-coherent propagation in thick layers, In: Photonics, Devices and Systems II, M. Hrabovský, D. Seuderáková and P. Tománek, eds., Proc. of SPIE 335-340 (2003), in print.

[6] O. Životský, Light propagation in thick magneto-optic layers, master thesis (2003), in Czech.

[7] K. Postava and T. Yamaguchi, and R. Kantor, Matrix description of coherent and incoherent light reflection and transmission by anisotropic multilayer structures, Appl. Opt. 41 2521-2531 (2002).

[8] D. M. Roessler, D. R. Huffman, Magnesium Oxide (MgO) in Handbook of Optical Solids II, Ed. E. D. Palík, Academic Press, New York (1991).

[9] C. Johnson and P. Christy, Phys. Rev. B 9, 5056 (1974), and Š. Višňovský et al., J. Magn. Mater. 127, 135 (1993).